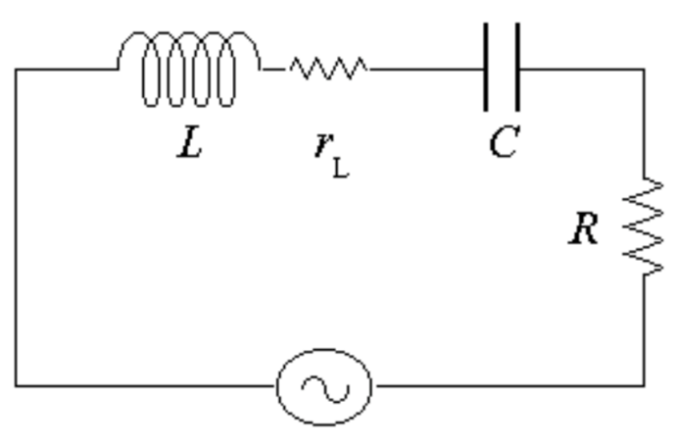
مدارهای RLC و پدیده تشدید



مدار RLC

$$V = V_R + V_C + V_L$$

$$i = i_m \sin \omega t$$

$$V = V_m \sin(\omega t + \varphi)$$

$$V_R = iR = i_m R \sin \omega t$$

$$R_0 = r_L + R$$

$$V_{R_0} = iR_0 = i_m R_0 \sin \omega t$$

$$V_C = \frac{q}{C} = \frac{\int i dt}{C} = \frac{i_m \int \sin \omega t \, dt}{C}$$

$$V_L = L\frac{di}{dt} = i_m L\omega \cos \omega t$$

$$V_{R_0} = iR_0 = i_m R_0 \sin \omega t$$

$$V_C = \frac{q}{C} = \frac{\int idt}{C} = \frac{i_m \int \sin \omega t \, dt}{C} = -\frac{i_m \cos \omega t}{C\omega} = i_m \frac{1}{C\omega} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$X_C = \frac{1}{C\omega}$$

مقاومت ظاهری خازن

$$V_L = L\frac{di}{dt} = i_m L\omega \cos \omega t = i_m L\omega \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$X_L = L\omega$$

مقاومت ظاهری سلف

$$V = V_m \sin(\omega t + \varphi) = V_{R_0} + V_C + V_L$$
$$= i_m R_0 \sin \omega t - i_m \frac{1}{C\omega} \cos \omega t + i_m L\omega \cos \omega t$$

$$V_m \cos(\varphi) \sin(\omega t) + V_m \sin(\varphi) \cos(\omega t)$$
$$= i_m R_0 \sin \omega t - i_m X_C \cos \omega t + i_m X_L \cos \omega t$$

$$V_m \cos(\varphi) \sin(\omega t) + V_m \sin(\varphi) \cos(\omega t)$$
$$= i_m R_0 \sin \omega t + i_m (X_L - X_C) \cos \omega t$$

$$V_m \cos(\varphi) = i_m R_0$$

$$V_m \sin(\varphi) = i_m (X_L - X_C)$$

$$\frac{V_m \sin(\varphi)}{V_m \cos(\varphi)} = \frac{i_m (X_L - X_C)}{i_m R_0}$$

$$\tan \varphi = \frac{V_{L,m} - V_{C,m}}{V_{R\,0,m}}$$
$$= \frac{X_L - X_C}{R_0}$$

$$V_m \cos\varphi = i_m R_0$$

$$V_m \sin\varphi = i_m (X_L - X_C)$$

$$V_m^2 \cos^2\varphi = i_m^2 R_o^2$$

$$V_m^2 \sin^2\varphi = i_m^2 (X_L - X_C)^2$$

$$V_m^2(\sin^2\varphi + \cos^2\varphi) = i_m^2 R_0^2 + i_m^2 (X_L - X_C)^2$$

$$V_m^2 = i_m^2 [R_0^2 + (X_L - X_C)^2]$$

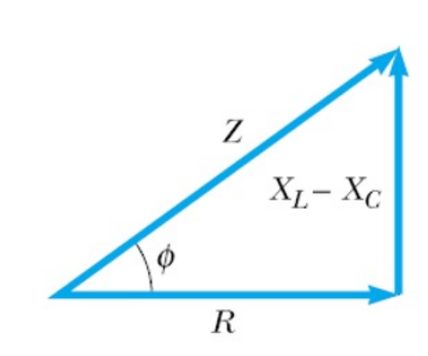
$$V_m = i_m \sqrt{R_0^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R_0^2 + (X_L - X_C)^2}$$

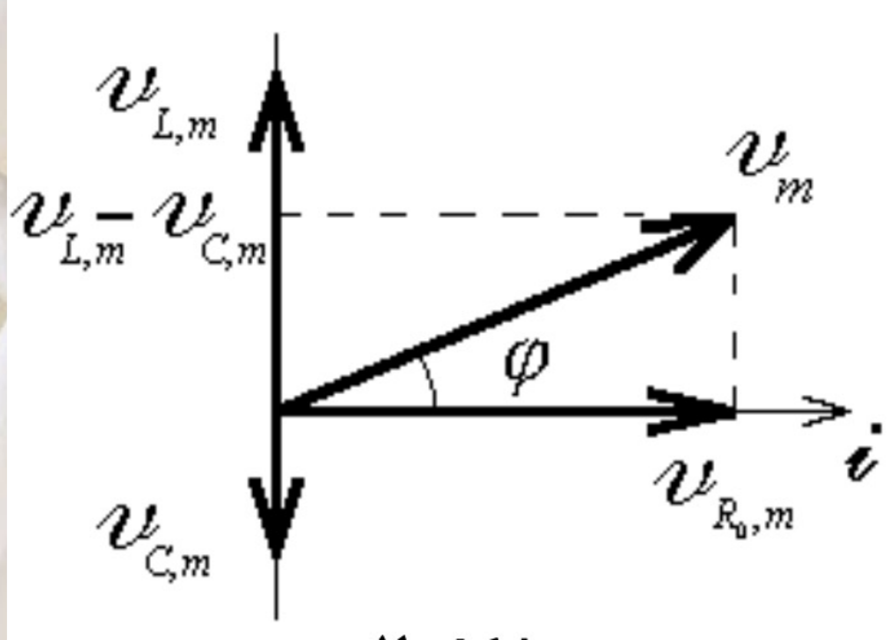
$$V_m = i_m Z$$

$$\tan\varphi = \frac{V_{L,m} - V_{C,m}}{V_{R\,0,m}} = \frac{X_L - X_C}{R_0}$$

مقاومت ظاهری مدارRLC



$$Z = \sqrt{R_0^2 + (X_L - X_C)^2}$$



$$V_m^2 = i_m^2[R_0^2 + (X_L - X_C)^2]$$

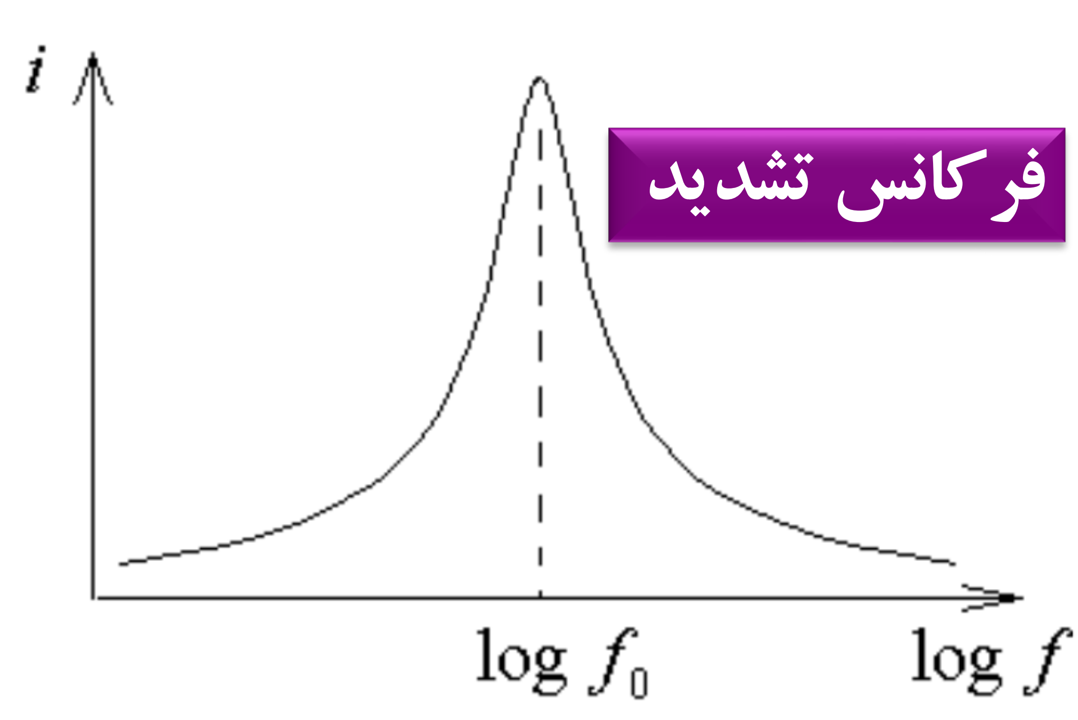
$$V_m^2 = i_m^2 R_0^2 + (i_m X_L - i_m X_C)^2$$

$$V_m^2 = V_{R_0,m}^2 + \left(V_{L,m} - V_{C,m}\right)^2$$

$$V_{R_0} = iR_0 = i_m R_0 \sin\omega t = V_{R_0,m} \sin\omega t$$

$$V_C = -\frac{i_m \cos\omega t}{C\omega} = i_m \frac{1}{C\omega} \sin\left(\omega t - \frac{\pi}{2}\right) = V_{C,m} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$V_L = i_m L\omega \cos\omega t = i_m L\omega \sin\left(\omega t + \frac{\pi}{2}\right) = V_{L,m} \sin\left(\omega t + \frac{\pi}{2}\right)$$



فرکانس تشدید

$$X_C = \frac{1}{C\omega}$$

$$X_L = L\omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

If $X_L = X_C$

$$Z = \sqrt{R^2} = R$$

$$\frac{1}{C\omega_0} = L\omega_0$$

$$LC\omega_0^2 = 1$$

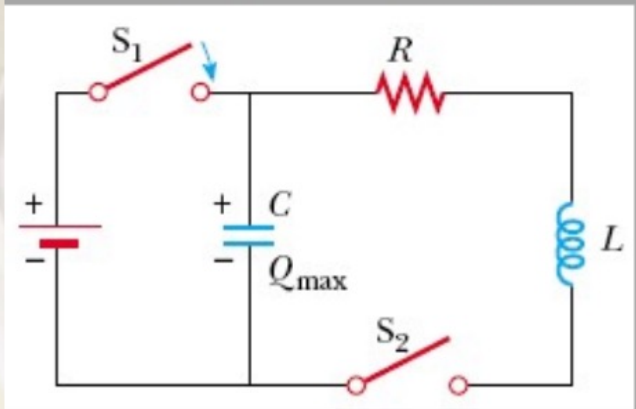
$$\omega_0^2 = \frac{1}{LC}$$

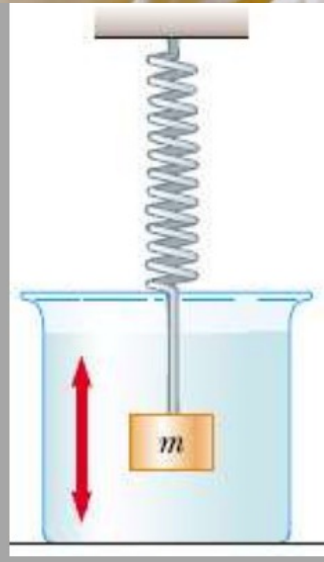
$$\omega_0 = 1/\sqrt{LC}$$

$$\omega_0 = 2\pi f_0$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The *RLC* Circuit





Analogies Between Electrical and Mechanical Systems

Electric Circuit		One-Dimensional Mechanical System
Charge	$Q \leftrightarrow x$	Position
Current	$I \leftrightarrow v_x$	Velocity
Potential difference	$\Delta V \leftrightarrow F_x$	Force
Resistance	$R \leftrightarrow b$	Viscous damping coefficient
Capacitance	$C \leftrightarrow 1/k$	(k = spring constant)
Inductance	$L \leftrightarrow m$	Mass
Current = time derivative of charge	$I = \frac{dQ}{dt} \leftrightarrow v_x = \frac{dx}{dt}$	Velocity = time derivative of position
Rate of change of current = second time derivative of charge	$\frac{dI}{dt} = \frac{d^2Q}{dt^2} \leftrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	Acceleration = second time derivative of position
Energy in inductor	$U_L = \frac{1}{2}LI^2 \leftrightarrow K = \frac{1}{2}mv^2$	Kinetic energy of moving object
Energy in capacitor	$U_C = \frac{1}{2}\frac{Q^2}{C} \leftrightarrow U = \frac{1}{2}kx^2$	Potential energy stored in a spring
Rate of energy loss due to resistance	$I^2R \leftrightarrow bv^2$	Rate of energy loss due to friction
RLC circuit	$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \leftrightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$	Damped object on a spring