General Form of Ampere's Law

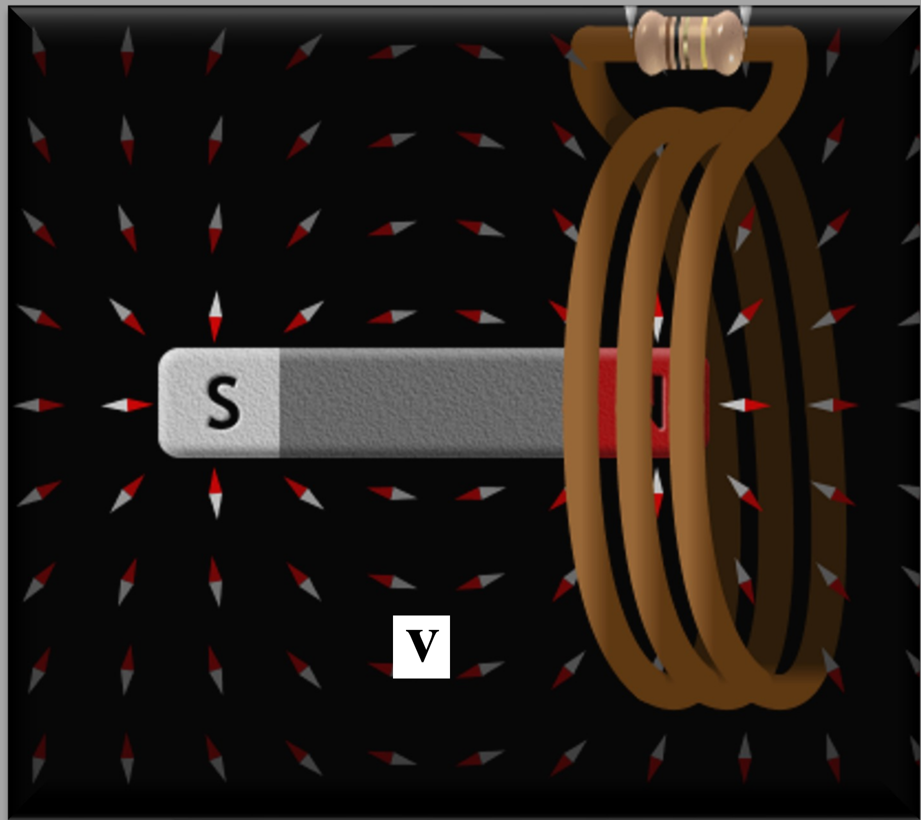
- In general Ampere's Law (Ampere-Maxwell's Law) should be

$$\boxed{\begin{aligned}\oint_{\text{loop}} \mathbf{B}\cdot d\mathbf{s} &= \mu_0\left(I_{\text{thru}} + I_{\text{disp}}\right)\\ &= \mu_0 I_{\text{thru}} + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}\end{aligned}}$$

- This shows that variation of E can generate B.
- Now we ask ourselves that whether variation of B with respect to time can generate E.
- The answer will be the subject of our next discussion.

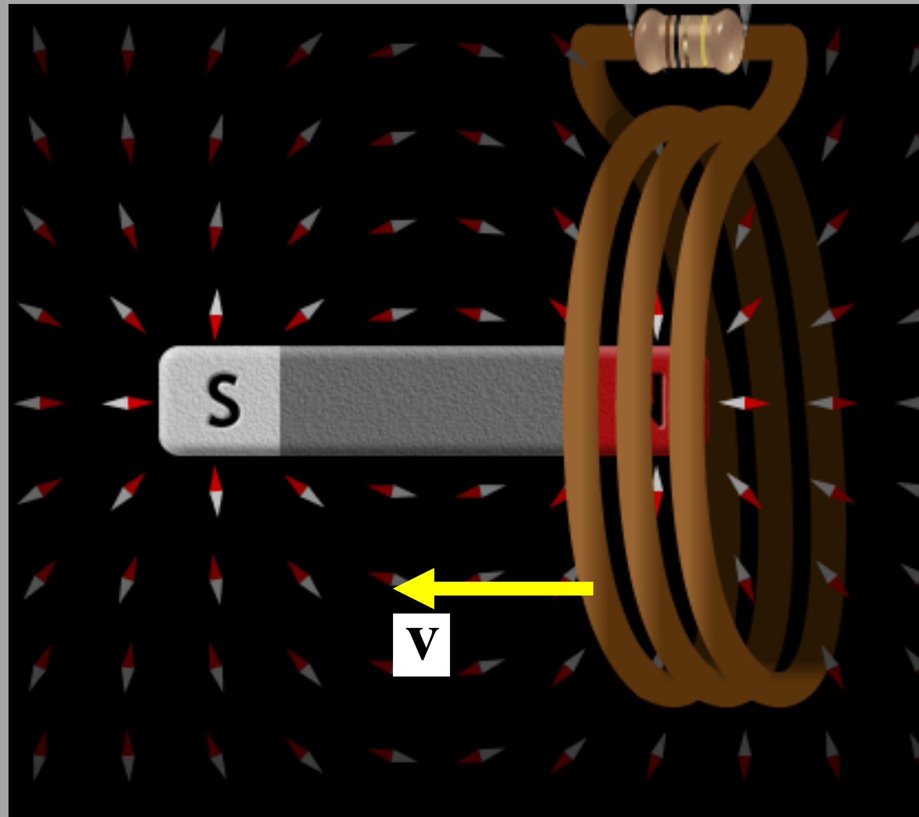
Moving Circuit in a Magnetic Field

- Imagine passing a wire loop around a magnet



Moving Circuit in a Magnetic Field

- Imagine passing a wire loop around a magnet
  - At the position pictured, the magnetic field is pointing approximately radially outward at the loop
  - What's the force on the electrons in the loop?

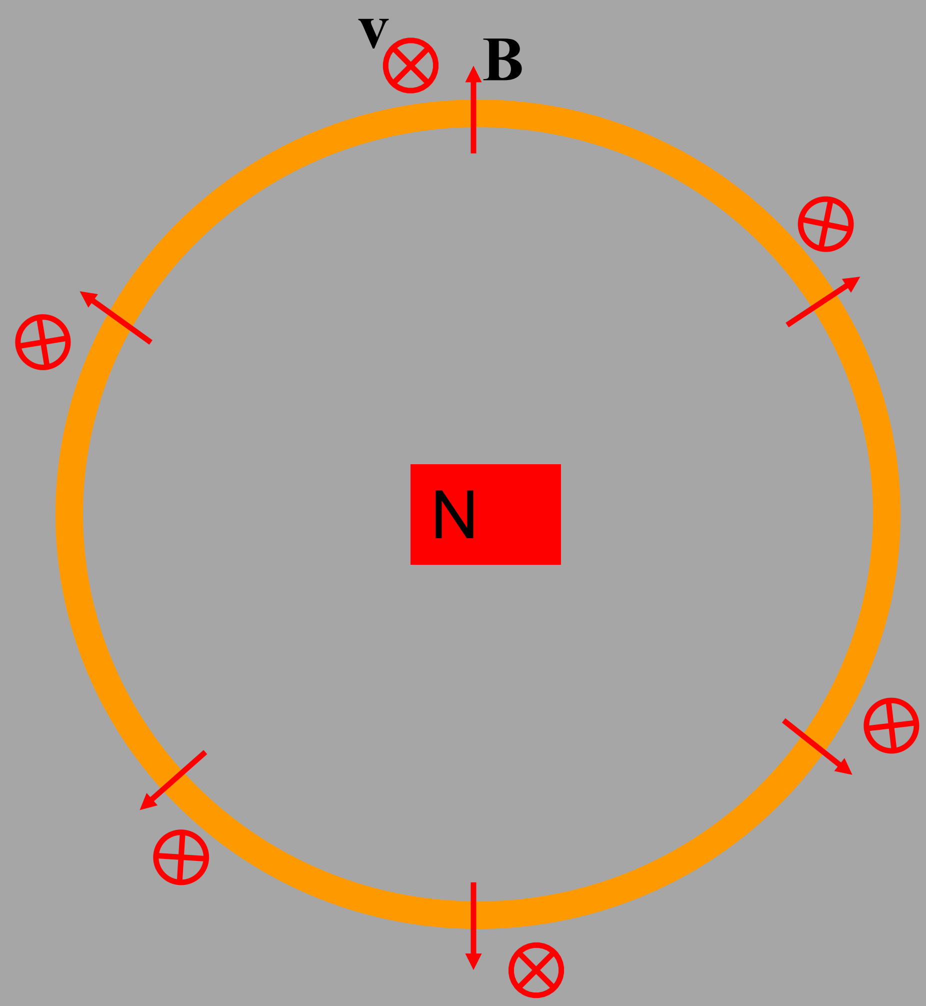


Moving Circuit in a Magnetic Field

- Imagine passing a wire loop around a magnet
  - At the position pictured, the magnetic field is pointing approximately radially outward at the loop
  - What's the force on the electrons in the loop?

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

$$|\mathbf{F}| = vB$$



Moving Circuit in a Magnetic Field

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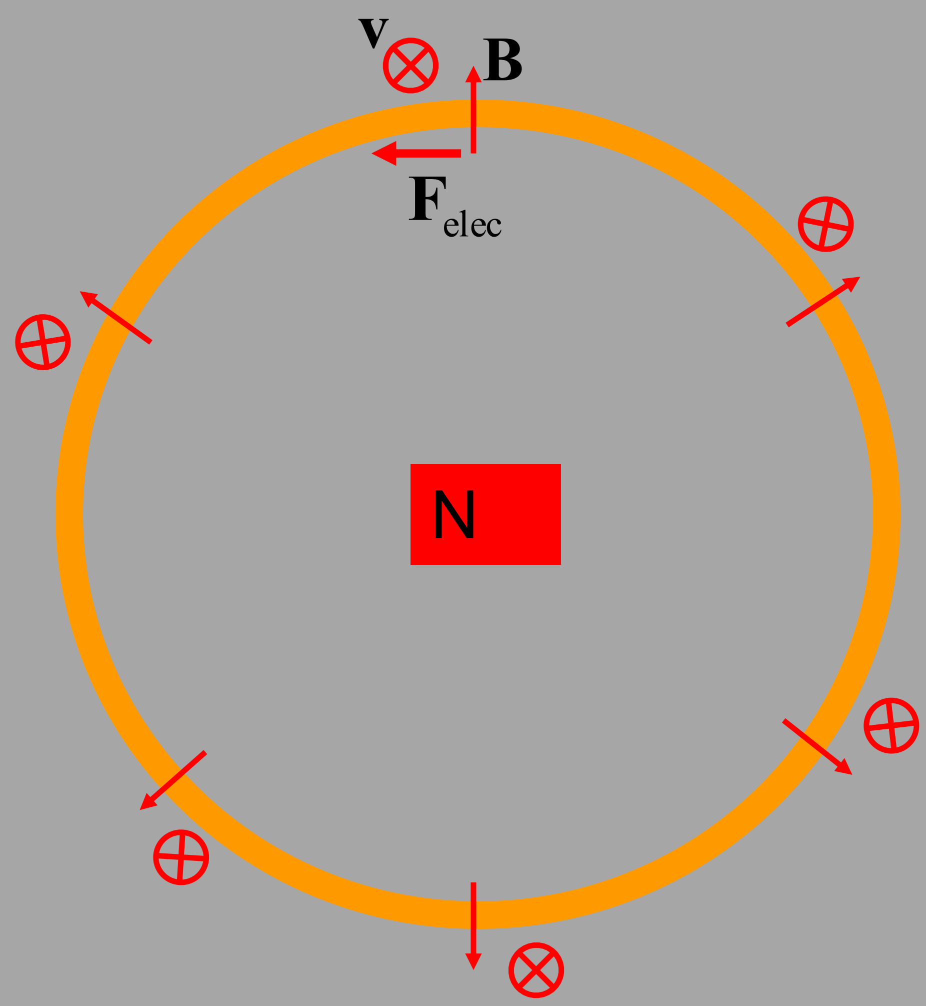
$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

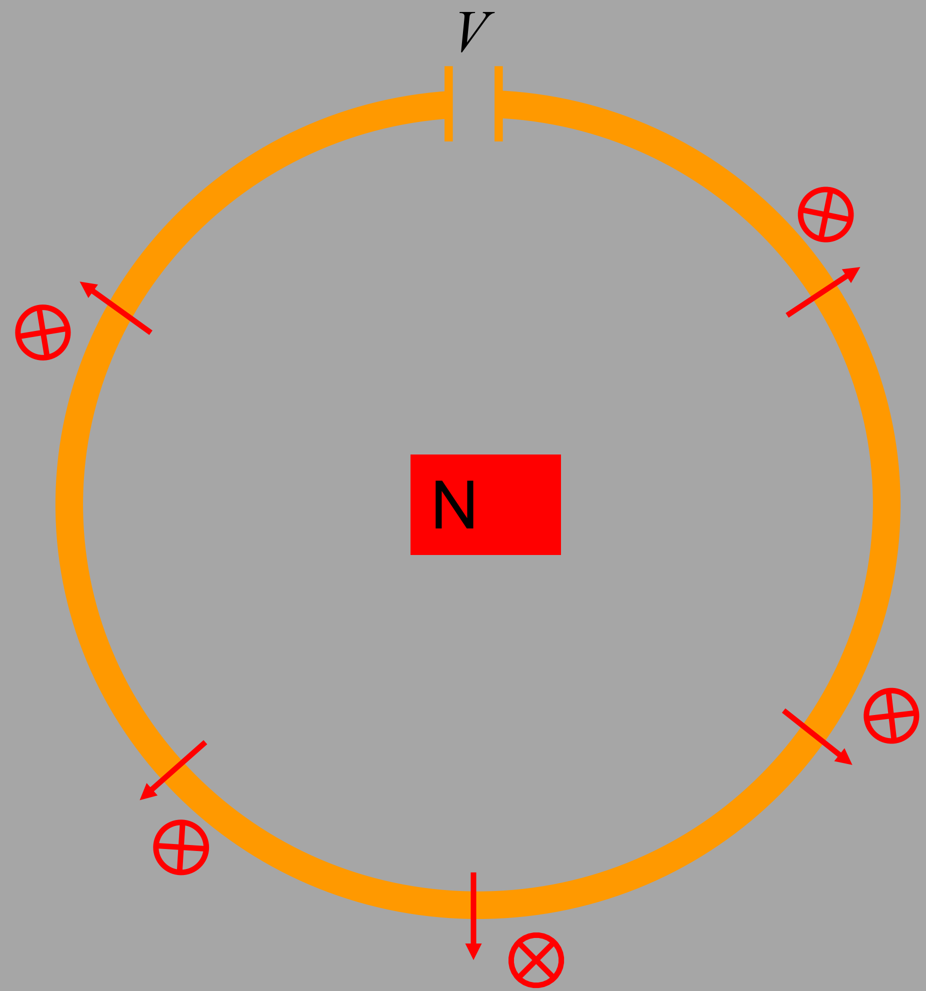
$$\left|\mathbf{F}\right| = evB$$

  - The force will start a current, like an **E** field!



Moving Circuit in a Magnetic Field

- Imagine passing a wire loop around a magnet
  - What's the induced "voltage"?



Moving Circuit in a Magnetic Field

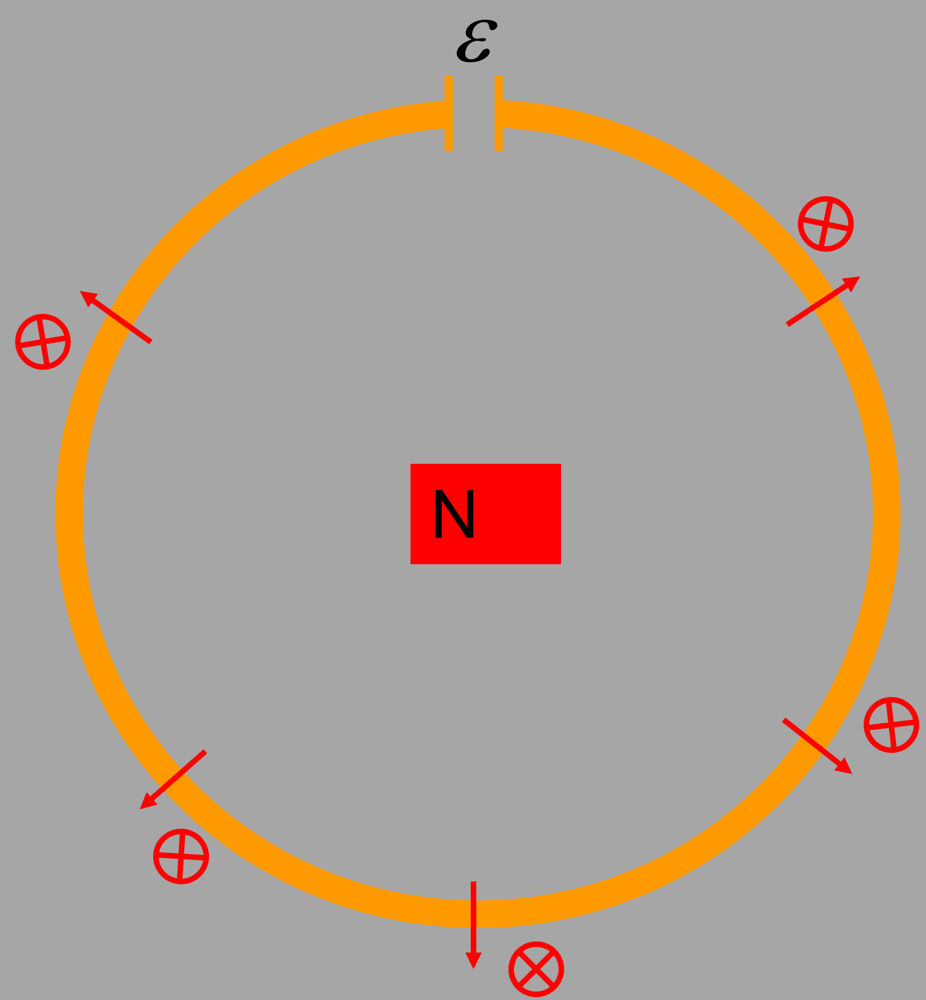
- Imagine passing a wire loop around a magnet
  - What's the induced "voltage" ?
  - The voltage is just the change in energy of a charge going around the loop

$$|\mathbf{F}| = qvB$$

$$V = \frac{1}{q}\int \mathbf{F}\cdot d\mathbf{s} = vBl = \mathcal{E}$$

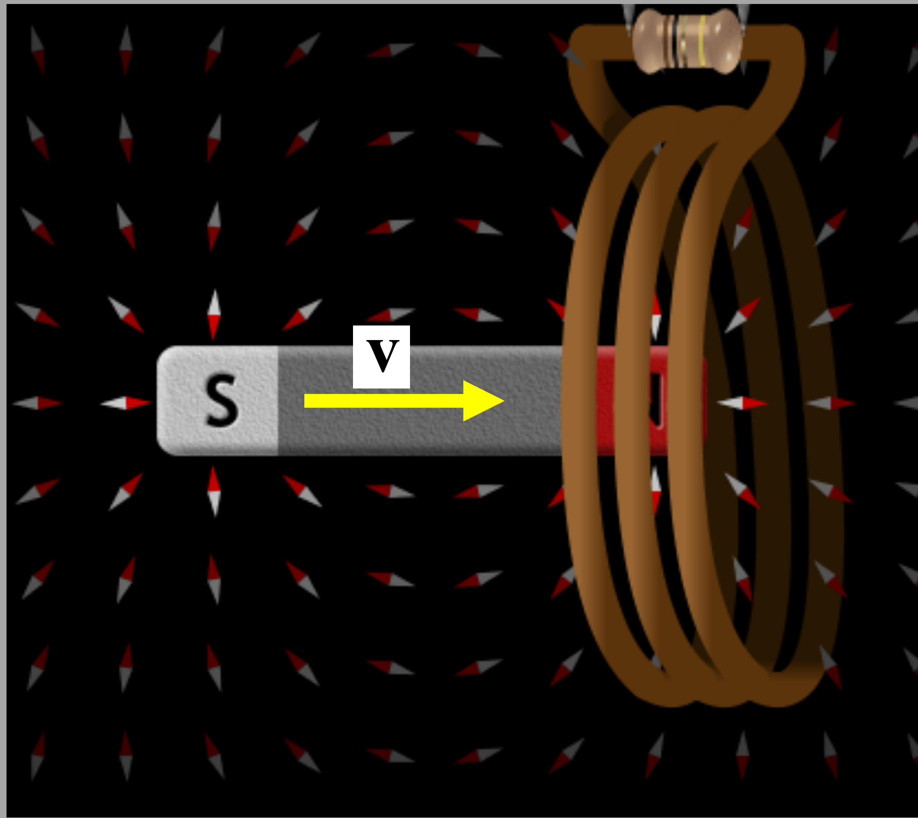
  - Call this voltage the "electromotive force" (EMF $\mathcal{E}$)

N.B. It's a voltage not a force!



Moving Circuit in a Magnetic Field

- Now imagine passing the bar magnet through the loop
  - It's the same situation, but different frame of reference.
  - The result is the same, but now it's the *changing* **B** field that causes the EMF



Moving Circuit in a Magnetic Field

- Consider another moving circuit
  - Again, **B** is constant and perpendicular to **v**

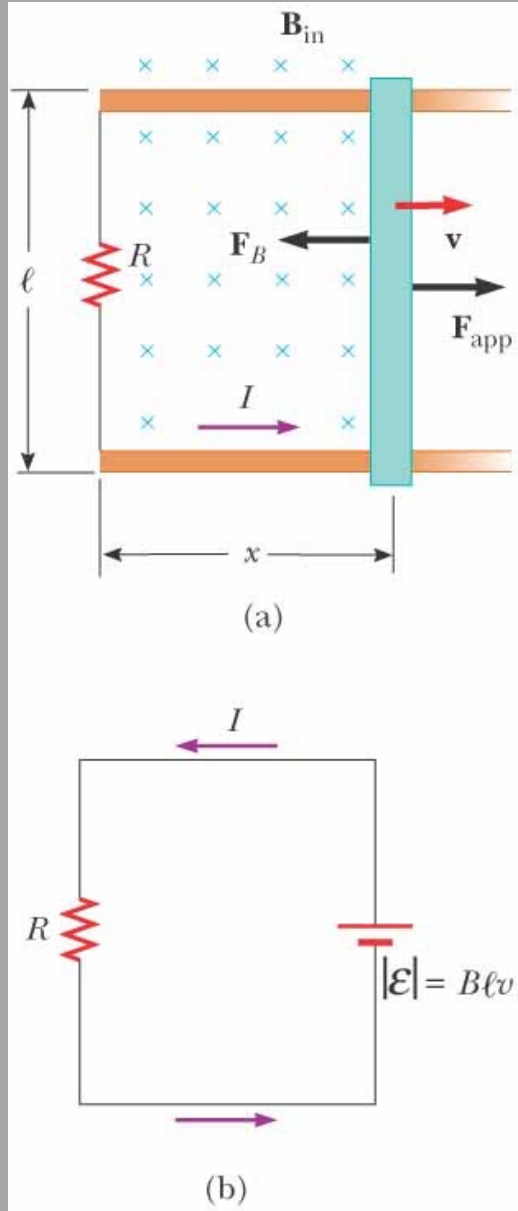
$$\left|\mathbf{F}\right| = qvB$$

$$\mathcal{E} = \frac{1}{q}\int \mathbf{F}\cdot d\mathbf{s} = vBl$$

  - What's the magnetic flux through the circuit?

$$\Phi_B = \oint \mathbf{B}\cdot d\mathbf{A} = BA = xBl$$

$$\frac{d\Phi_B}{dt} = \frac{dx}{dt}Bl = vBl = \pm\mathcal{E}$$



Moving Circuit in a Magnetic Field

- Consider another moving circuit
  - What's the current through the resistor?

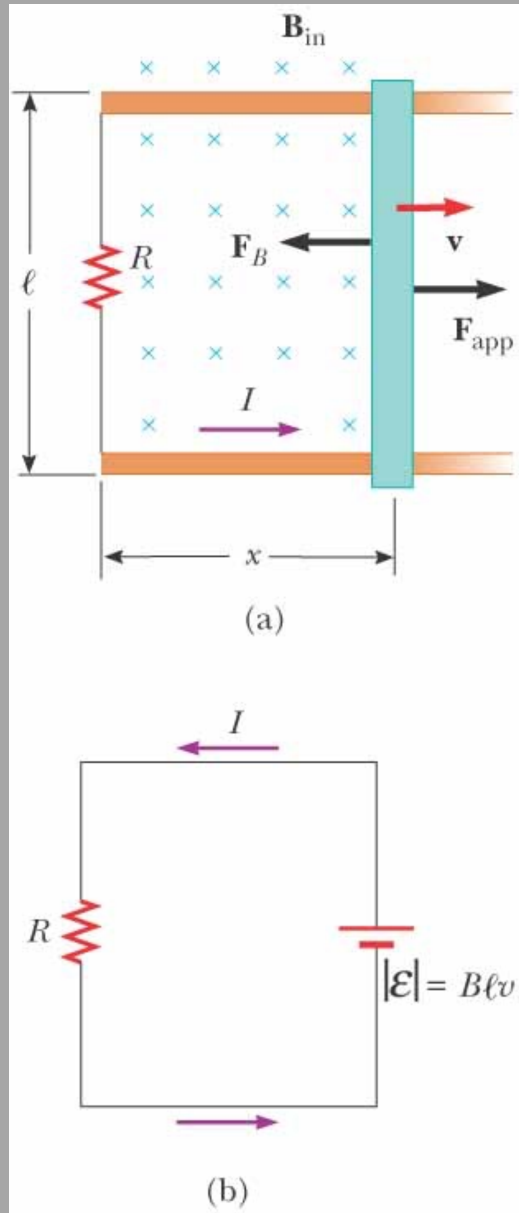
$$I = \frac{|\mathcal{E}|}{R} = \frac{vBl}{R}$$

  - What's the force on this current?

$$F_I = |l\mathbf{I} \times \mathbf{B}| = lIB$$

  - For a constant velocity, what power must applied

$$P = \mathbf{F} \cdot \mathbf{v} = (IlB)v = \frac{lBv}{R} lBv = \frac{\mathcal{E}^2}{R}$$



Faraday's Law

- Putting together what we've learned we can state Faraday's Law

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

  - The negative sign tells which way the current will run in a loop…

How to Generate an EMF

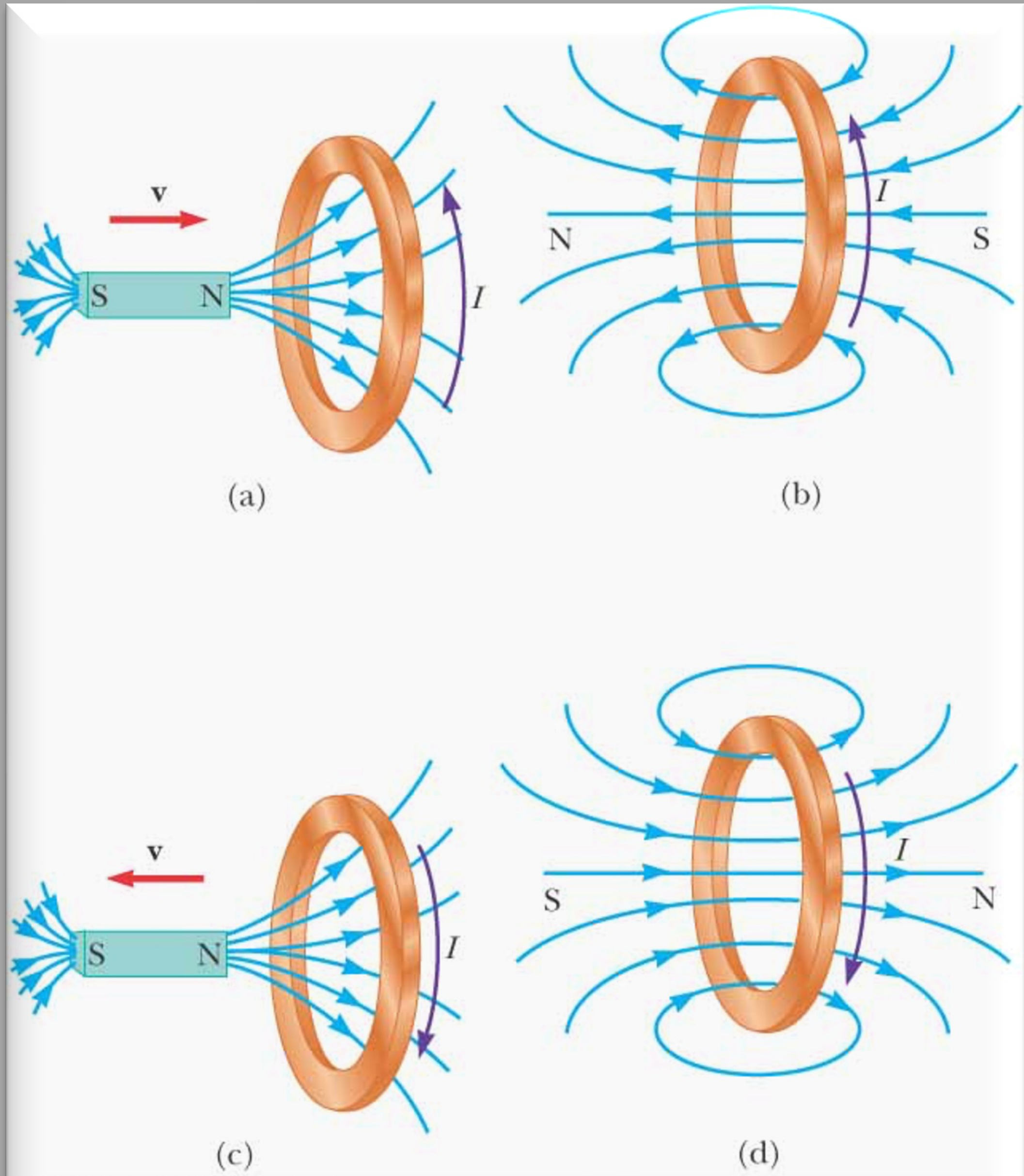
- There are several ways to generate a current in a wire loop with a magnetic field, without using a battery
  - Change the magnitude of **B** with time
  - Change the area of the loop
  - Change the orientation of the loop with respect to **B**
- All of these are covered by Faraday’s Law

Lenz’s Law

- Which way will the current flow?
- Lenz’s Law:
  – The induced current in a loop will create a magnetic field that *opposes* the change in the magnetic flux through the area of the loop

Example: Current in a Loop

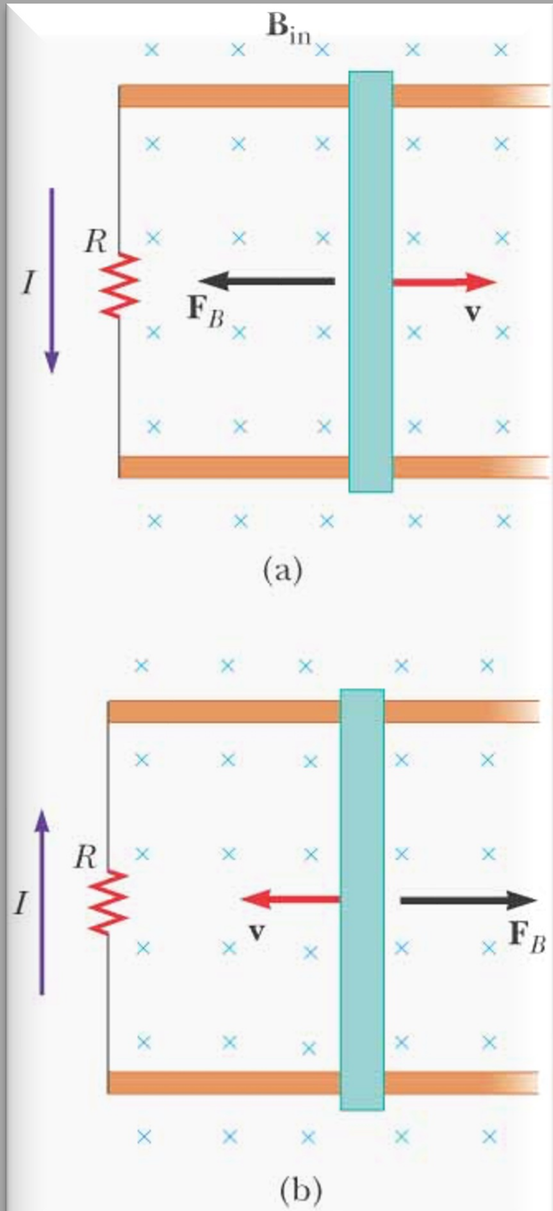
- If the magnet is approaching the loop, more field lines go through (to the right)
  - The induced *field* must be to the left
  - So the current must be counterclockwise
- If the magnet is going away
  - Fewer lines
  - Field to right
  - Clockwise



Example: Sliding Bar

- If the bar is sliding to the right
  - More field lines (into page)
  - Induced field out of page
  - Current counterclockwise

- If the bar is sliding to the left
  - Fewer field lines (into page)
  - Induced field into page
  - Current clockwise



Induced Electric FIeld

- Note that an EMF implies the existence of an electric field

$$\mathcal{E} = \frac{1}{q}\int \mathbf{F}\cdot d\mathbf{s} = \int \mathbf{E}\cdot d\mathbf{s}$$

  - This is a non-conservative electric field, unlike the electrostatic cases we've considered before

- We can thus write Faraday's Law, this way

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

- Or this way

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A}$$

The loop contains the area

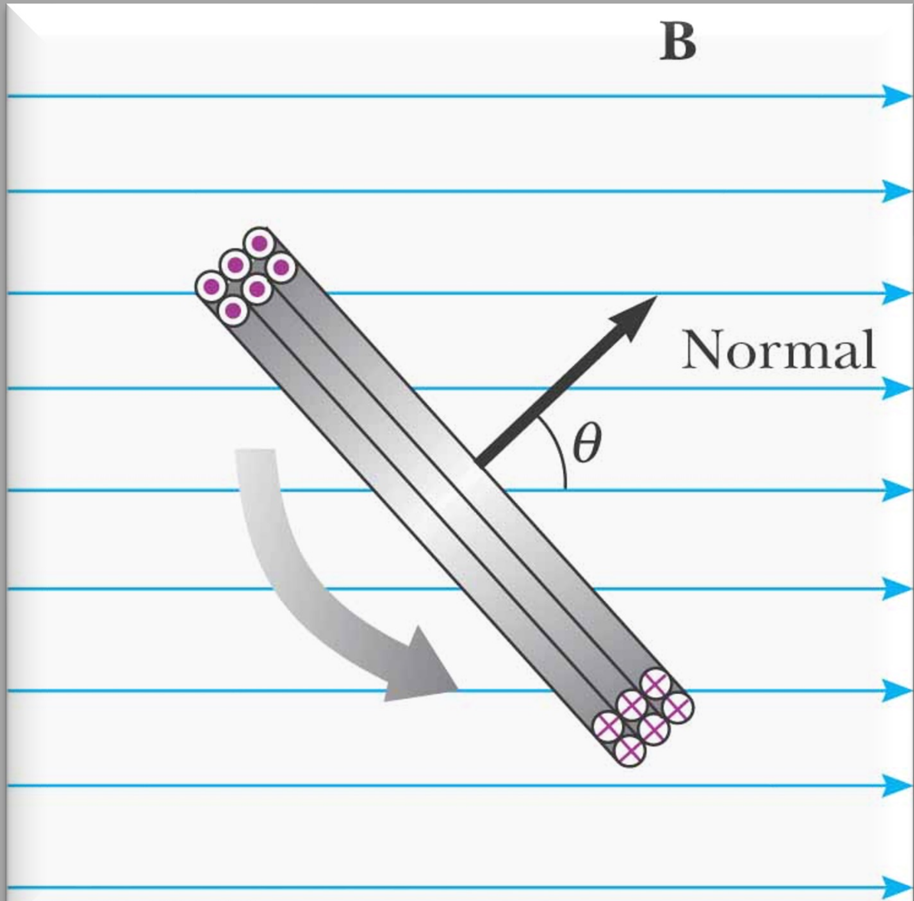
Generators and Motors

- A current loop, rotating in a constant magnetic field generates a varying EMF

$$\Phi_B = \mathbf{B}\cdot\mathbf{A} = AB\cos\omega t$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = NAB\omega\sin\omega t$$

N Turns in loop, each contribute



Generators and Motors

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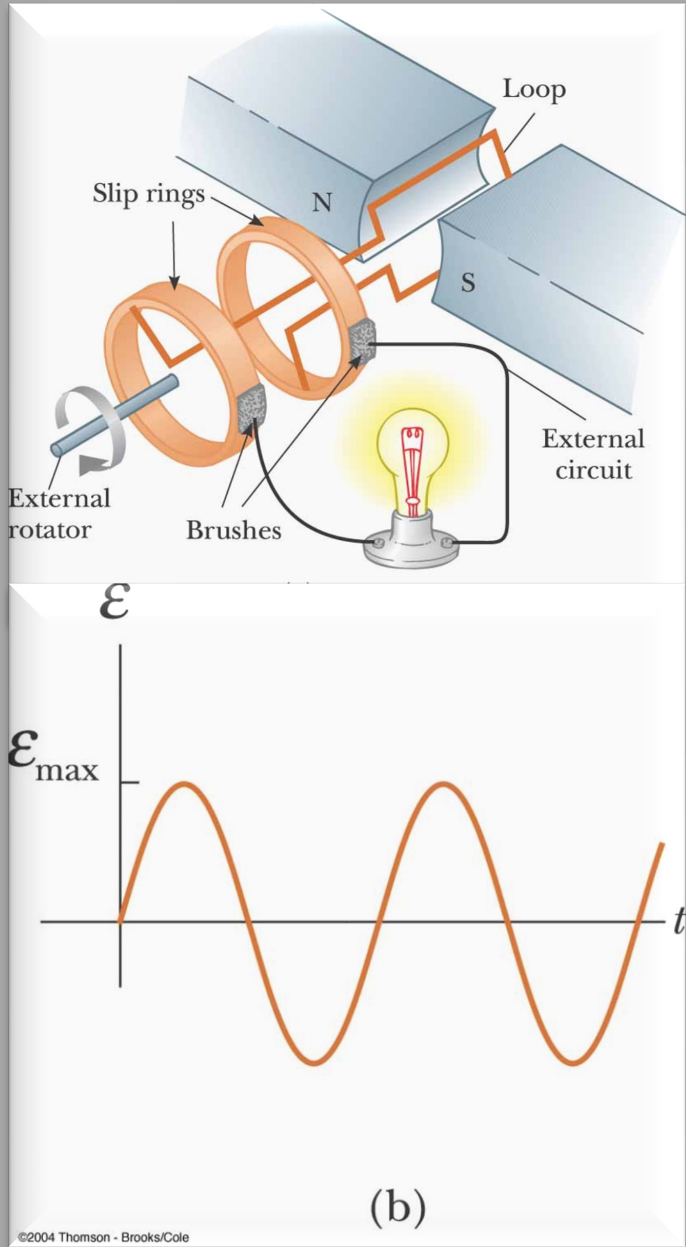
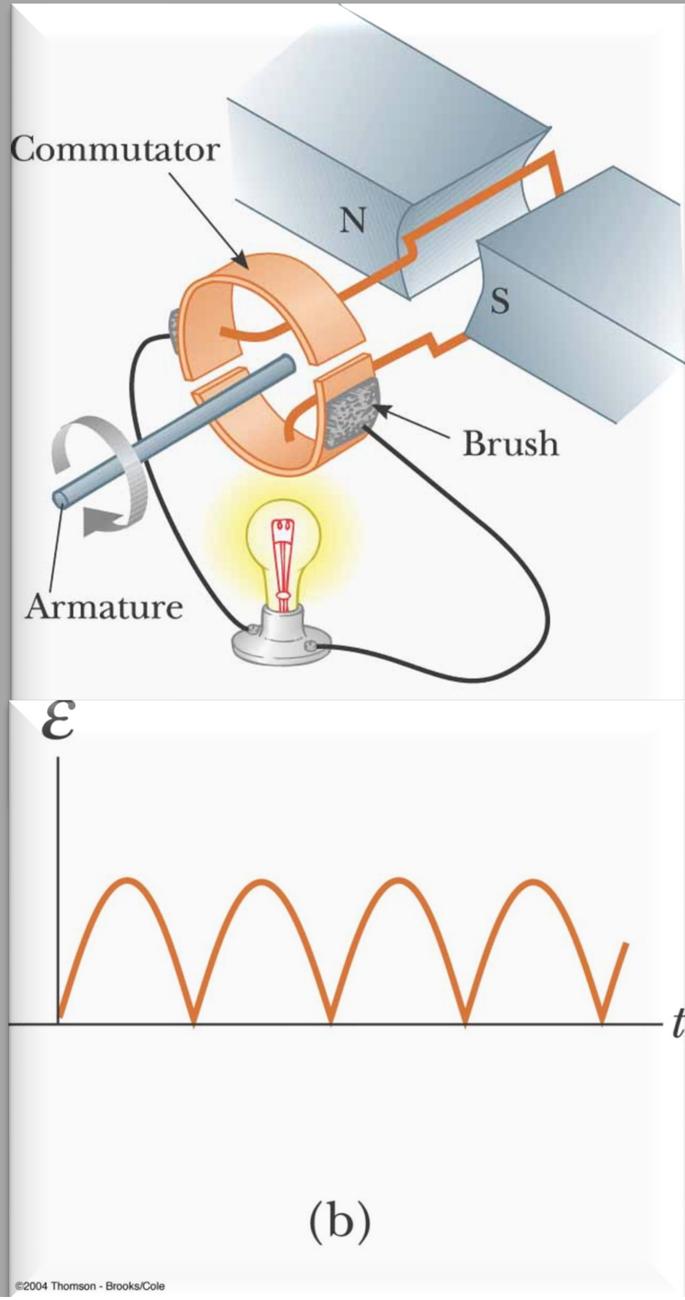
- This is an AC generator (ω = 2π*60 in US)



Generators and Motors

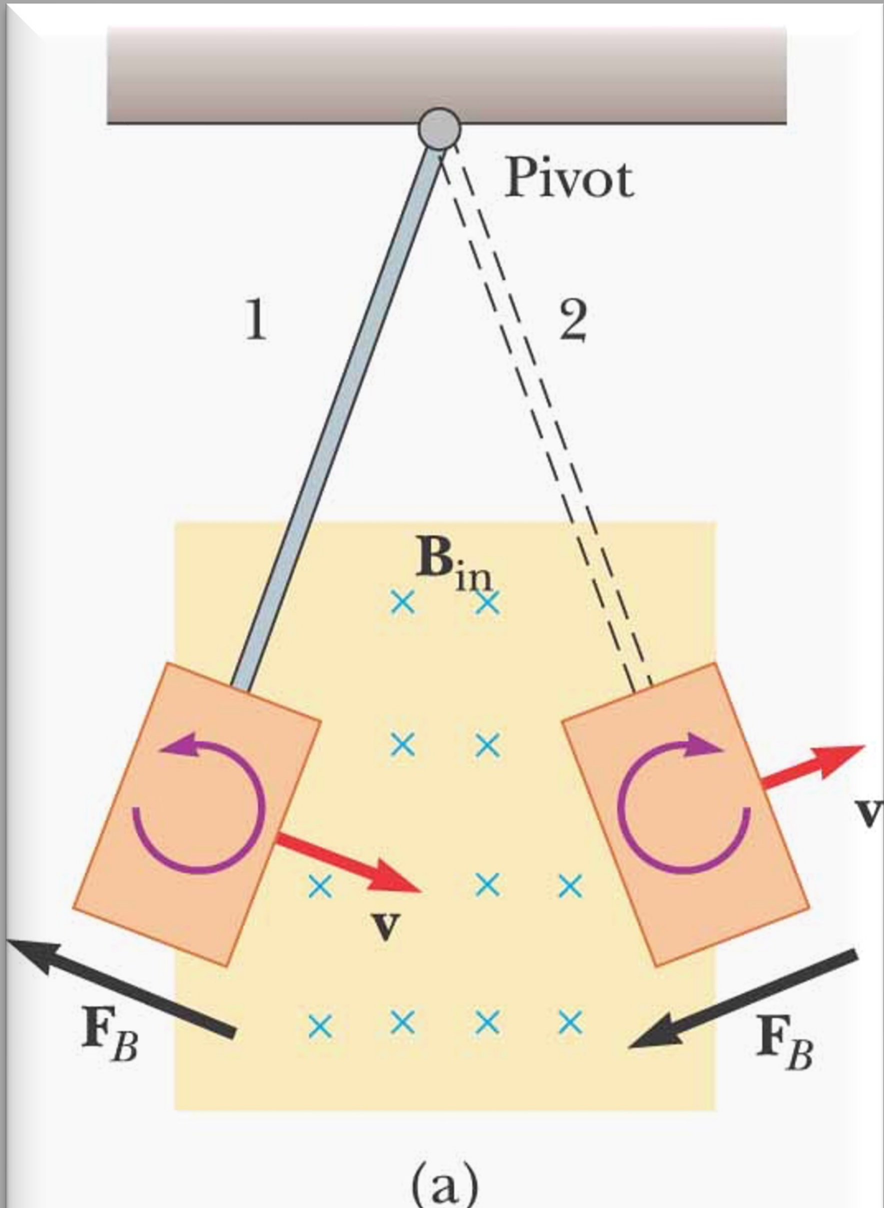
- Can make a DC generator, by switching leads every half turn
- By running a current through the wire (with a battery), can make the loop turn via the torque
  - This is a motor
- Current in: motor

  Current out: generator



Eddy Currents

- A conductor *moving* through a magnetic field will have induced currents
  - The currents will where the field is changing
  - The force from the magnetic field on these currents always work against the velocity



Eddy Currents

- A conductor *moving* through a magnetic field will have induced currents
  - The currents will where the field is changing
  - The force from the magnetic field on these currents always work against the velocity
  - Cutting strips into the conductor decreases the size of the eddies and thus the retarding force

Maxwell's Equations

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$