CHAPTER OUTLINE

32.1 Self-Inductance

32.2 *RL* Circuits

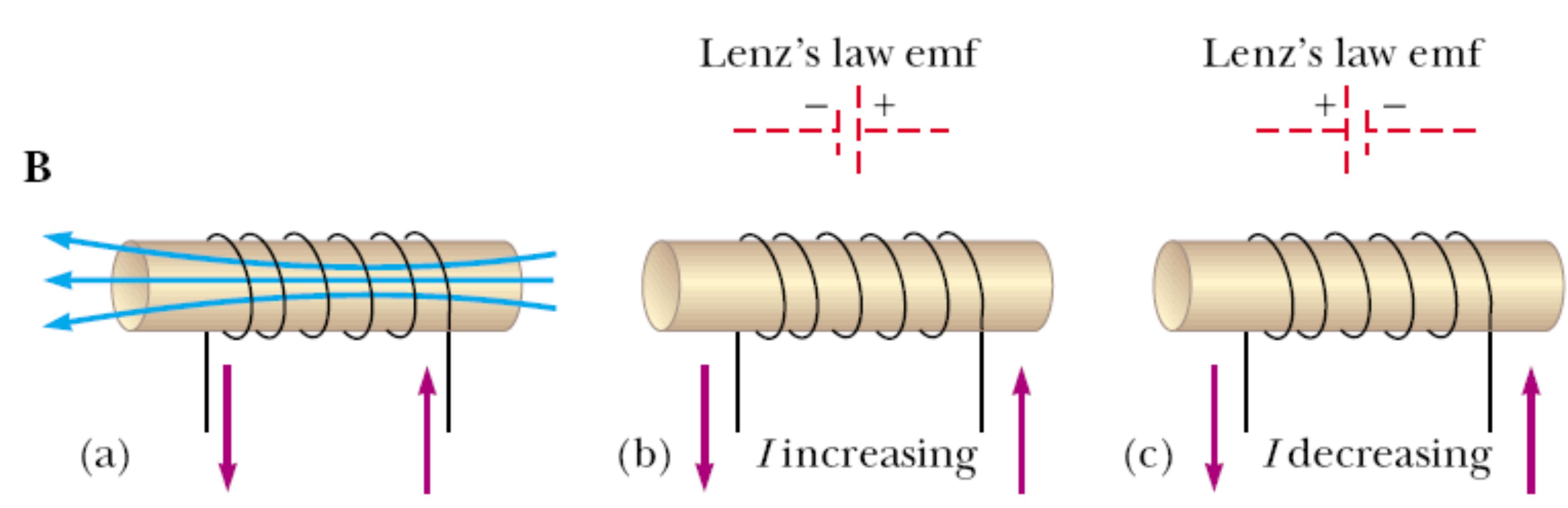
32.3 Energy in a Magnetic Field

32.4 Mutual Inductance

32.5 Oscillations in an *LC* Circuit

32.6 The *RLC* Circuit

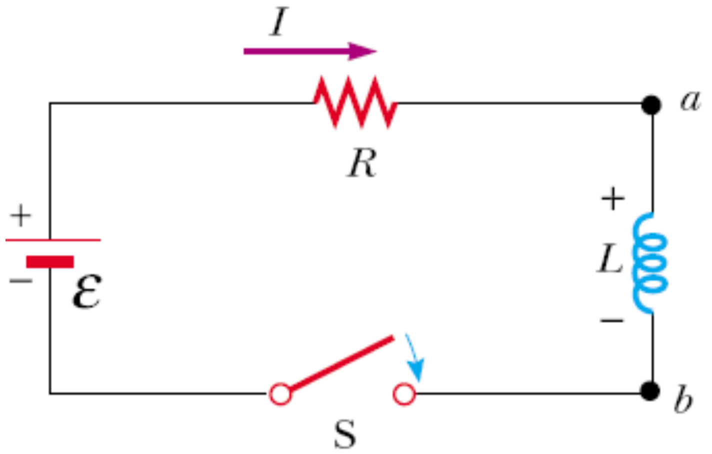
▲ *An airport metal detector contains a large coil of wire around the frame. This coil has a property called inductance. When a passenger carries metal through the detector, the inductance of the coil changes, and the change in inductance signals an alarm to sound. (Jack Hollingsworth/Getty Images)*



(a) A current in the coil produces a magnetic field directed to the left. (b) If the current increases, the increasing magnetic flux creates an induced emf in the coil having the polarity shown by the dashed battery. (c) The polarity of the induced emf reverses if the current decreases.

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

RL Circuits



$$\varepsilon - V_R - V_L = 0$$

$$\varepsilon - RI - L\frac{dI}{dt} = 0$$

$$\frac{dI}{dt} = -\frac{RI - \varepsilon}{L} = -\frac{I - \varepsilon/R}{L/R}$$

$$\frac{dI}{I - \varepsilon/R} = -\frac{dt}{L/R}$$

$$\int \frac{dI}{I - \varepsilon/R} = -\frac{R}{L}\int dt$$

$$\ln(I - \varepsilon/R) = -\frac{R}{L}t + \ln K$$

$$t = 0 \Rightarrow I = 0$$

$$\ln(0 - \varepsilon/R) = -\frac{R}{L}\times 0 + \ln K = \ln K$$

$$\ln(-\varepsilon/R) = \ln K$$

$$\ln(I - \varepsilon/R) = -\frac{R}{L}t + \ln(-\varepsilon/R)$$

$$\ln\left(\frac{I - \frac{\varepsilon}{R}}{-\varepsilon/R}\right) = -\frac{R}{L}t$$

$$\frac{I - \frac{\varepsilon}{R}}{-\varepsilon/R} = \exp\left(-\frac{Rt}{L}\right)$$

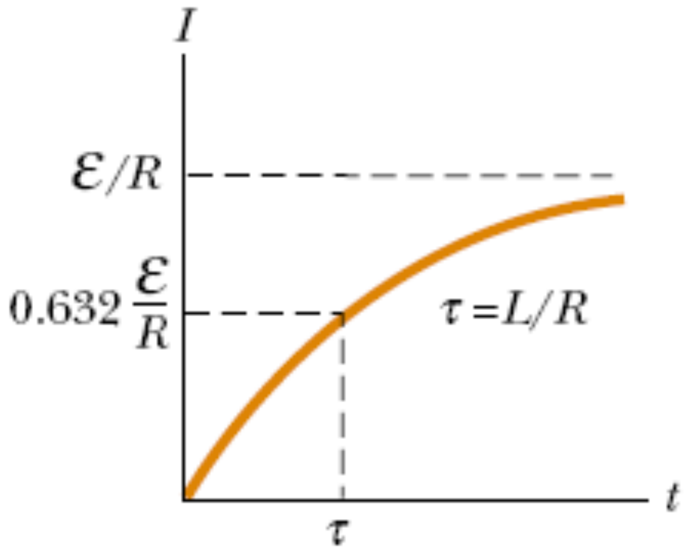
$$\boxed{\frac{I-\frac{\varepsilon}{R}}{-\varepsilon/R}=\exp\left(-\frac{Rt}{L}\right)}$$

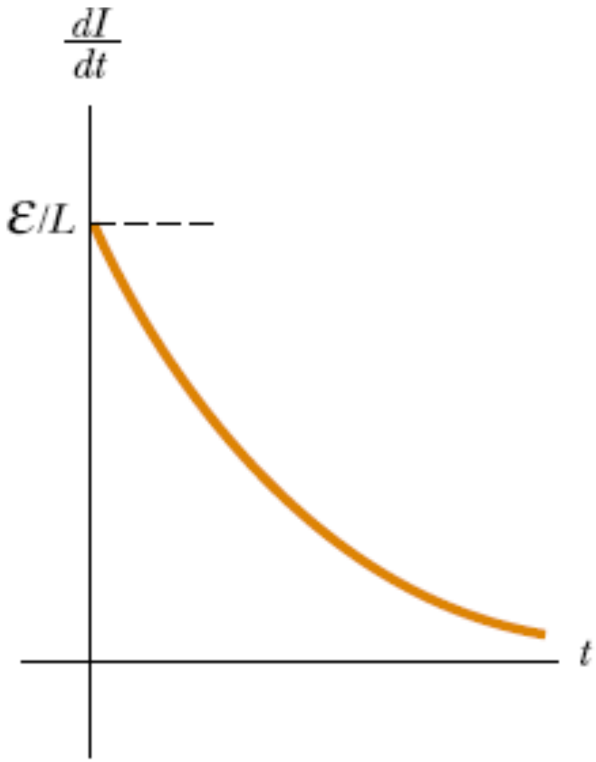
$$\boxed{I-\frac{\varepsilon}{R}=-\varepsilon/R\exp\left(-\frac{Rt}{L}\right)}$$

$$\boxed{I=\frac{\varepsilon}{R}\left(1-\exp\left(-\frac{Rt}{L}\right)\right)}$$

$$\boxed{\frac{dI}{dt}=\frac{\varepsilon}{R}\left(0-\frac{-R}{L}\exp\left(-\frac{Rt}{L}\right)\right)}$$

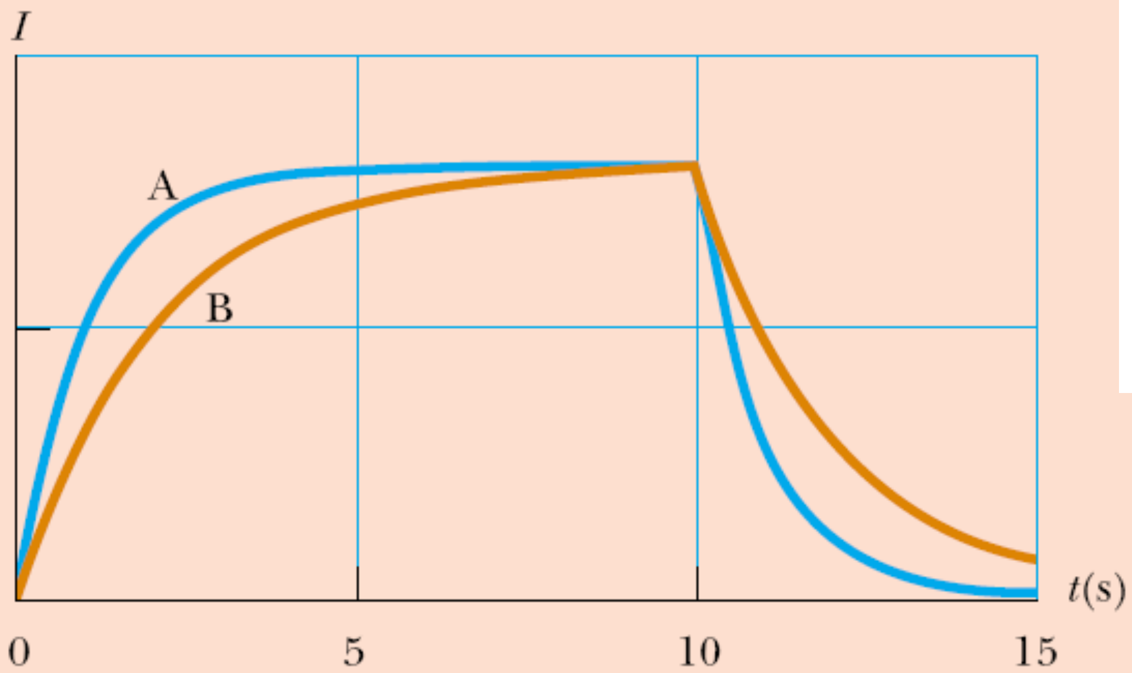
$$\boxed{\frac{dI}{dt}=\frac{\varepsilon}{L}\exp\left(-\frac{Rt}{L}\right)}$$

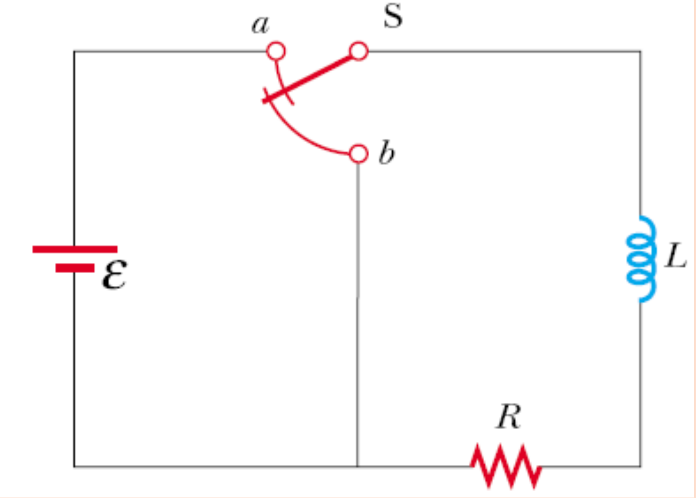




Quick Quiz 32.4 Two circuits like the one shown in Figure 32.6 are identical except for the value of L. In circuit A the inductance of the inductor is L_A, and in circuit B it is L_B. Switch S is thrown to position a at $t = 0$. At $t = 10$ s, the switch is thrown to position b. The resulting currents for the two circuits are as graphed in Figure 32.10.

If we assume that the time constant of each circuit is much less than 10 s, which of the following is true? (a) $L_A > L_B$; (b) $L_A < L_B$; (c) not enough information to tell.

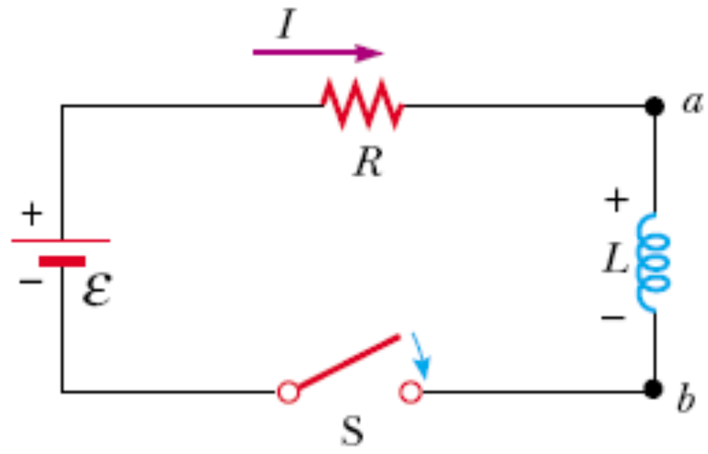




(b). Figure 32.10 shows that circuit B has the greater time constant because in this circuit it takes longer for the current to reach its maximum value and then longer for this current to decrease to zero after the switch is thrown to position b. Equation 32.8 indicates that, for equal resistances R_A and R_B, the condition $\tau_B > \tau_A$ means that $L_A < L_B$.

Energy in a Magnetic Field

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$



$$I\mathcal{E} = I^2R + LI\frac{dI}{dt}$$

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

$$U = \int dU = \int_0^I LI\,dI = L\int_0^I I\,dI$$

$$\boxed{U = \tfrac{1}{2}LI^2}$$

$$B = \mu_0 nI = \mu_0\frac{N}{\ell}I \qquad \Phi_B = BA = \mu_0\frac{NA}{\ell}I \qquad L = \frac{N\Phi_B}{I} = \boxed{\frac{\mu_0 N^2 A}{\ell}} \qquad L = \mu_0\frac{(n\ell)^2}{\ell}A = \mu_0 n^2 A\ell = \mu_0 n^2 V$$

$$B \propto I$$

$$\Phi \propto B$$

$$\Phi \propto I$$

$$\Phi = bI$$

$$\varepsilon_1 = -\frac{d\Phi}{dt}$$

$$\varepsilon_1 = -b\frac{dI}{dt}$$

$$\varepsilon_L = -N\varepsilon_1$$

$$\varepsilon_L = -Nb\frac{dI}{dt}$$

$$Nb = L$$

$$\varepsilon_L = -L\frac{dI}{dt}$$

$$B = \mu_0 \frac{NI}{l}$$

$$B = k\mu_0 \frac{N}{l} I$$

$$\Phi = AB = k\mu_0 \frac{NA}{l} I$$

$$\varepsilon_L = -N\frac{d\Phi}{dt}$$

$$\varepsilon_L = -k\mu_0 \frac{N^2 A}{l}\frac{dI}{dt}$$

$$\varepsilon_L = -L\frac{dI}{dt}$$

$$L = k\mu_0 \frac{N^2 A}{l}$$

$$-\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

$$L = \frac{d\Phi}{dI} = \frac{\Phi}{I}$$

$$L = \mu_0 \frac{(n\ell)^2}{\ell} A = \mu_0 n^2 A\ell = \mu_0 n^2 V$$

$$B = \mu_0 n I$$

$$U = \tfrac{1}{2} LI^2 = \tfrac{1}{2} \mu_0 n^2 A \ell \left(\frac{B}{\mu_0 n} \right)^2 = \frac{B^2}{2\mu_0} A\ell$$

$$u_B = \frac{U}{A\ell} = \frac{B^2}{2\mu_0}$$

Example 32.4 What Happens to the Energy in the Inductor?

Consider once again the *RL* circuit shown in Figure 32.6. Recall that the current in the right-hand loop decays exponentially with time according to the expression $I = I_0 e^{-t/\tau}$, where $I_0 = \mathcal{E}/R$ is the initial current in the circuit and $\tau = L/R$ is the time constant. Show that all the energy initially stored in the magnetic field of the inductor appears as internal energy in the resistor as the current decays to zero.

$$\frac{dU}{dt} = I^2R = (I_0e^{-Rt/L})^2R = I_0^{\,2}\,Re^{-2Rt/L}$$

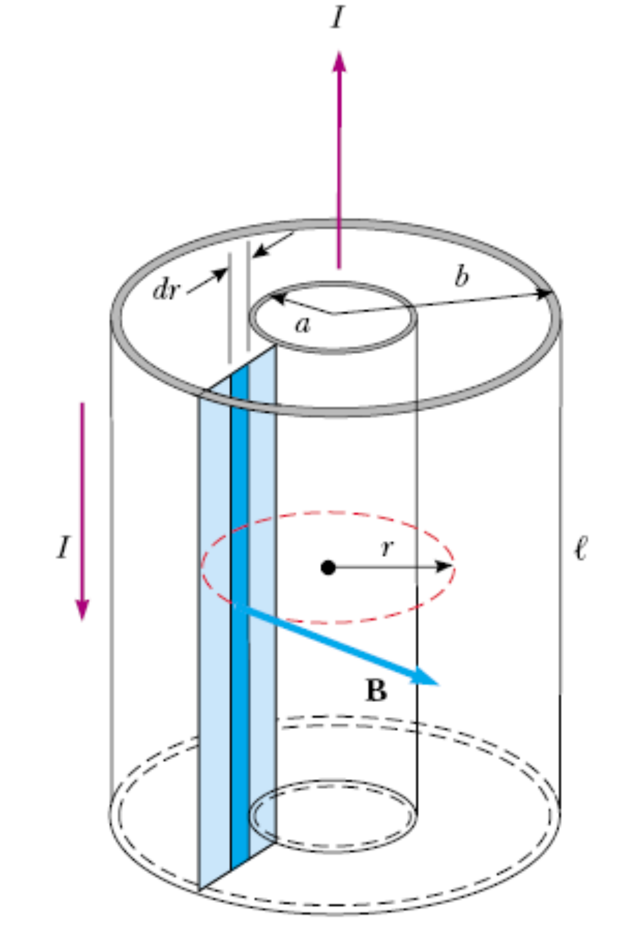
$$U = \int_0^\infty I_0^2\,Re^{-2Rt/L}\,dt = I_0^{\,2}R\int_0^\infty e^{-2Rt/L}\,dt$$

$$U = I_0^2\,R\left(\frac{L}{2R}\right) = \tfrac{1}{2}\,LI_0^{\,2}$$

Example 32.5 The Coaxial Cable

Coaxial cables are often used to connect electrical devices, such as your stereo system, and in receiving signals in TV cable systems. Model a long coaxial cable as consisting of two thin concentric cylindrical conducting shells of radii a and b and length ℓ, as in Figure 32.13. The conducting shells carry the same current I in opposite directions. Imagine that the inner conductor carries current to a device and that the outer one acts as a return path carrying the current back to the source.

(A) Calculate the self-inductance L of this cable.



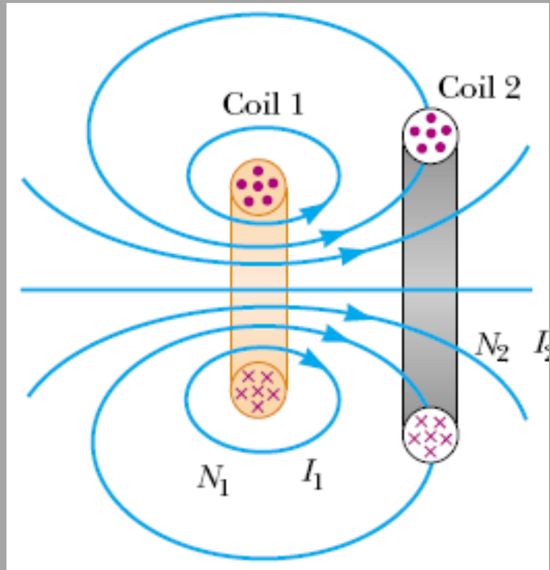
$$\Phi_B = \int B\, dA = \int_a^b \frac{\mu_0 I}{2\pi r}\, \ell\, dr = \frac{\mu_0 I \ell}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 I \ell}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$L = \frac{\Phi_B}{I} = \frac{\mu_0 \ell}{2\pi} \ln\left(\frac{b}{a}\right)$$

(B) Calculate the total energy stored in the magnetic field of the cable.

$$U = \tfrac{1}{2} L I^2 = \frac{\mu_0 \ell I^2}{4\pi} \ln\left(\frac{b}{a}\right)$$

Mutual Inductance



$$M_{12} \equiv \frac{N_2 \Phi_{12}}{I_1}$$

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{12}}{dt} = -N_2 \frac{d}{dt}\left(\frac{M_{12} I_1}{N_2}\right) = -M_{12} \frac{dI_1}{dt}$$

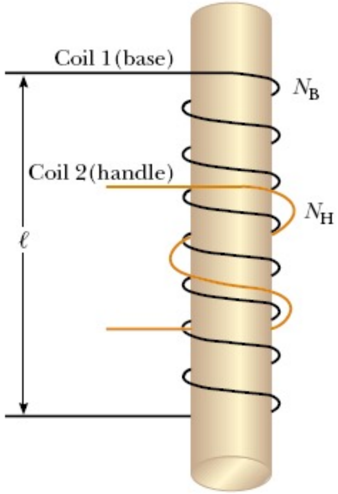
$$\mathcal{E}_1 = -M_{21} \frac{dI_2}{dt}$$

$$\mathcal{E}_2 = -M \frac{dI_1}{dt} \qquad \text{and} \qquad \mathcal{E}_1 = -M \frac{dI_2}{dt}$$

Example 32.6 "Wireless" Battery Charger

An electric toothbrush has a base designed to hold the toothbrush handle when not in use. As shown in Figure 32.15a, the handle has a cylindrical hole that fits loosely over a matching cylinder on the base. When the handle is placed on the base, a changing current in a solenoid inside the base cylinder induces a current in a coil inside the handle. This induced current charges the battery in the handle.

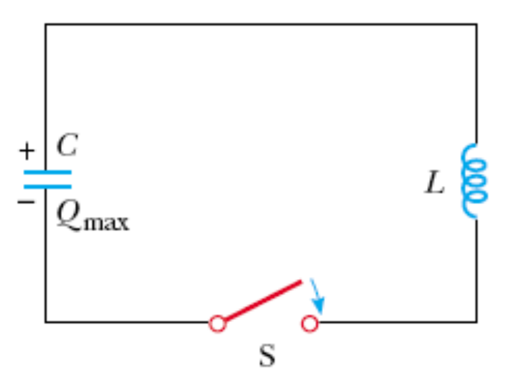
We can model the base as a solenoid of length ℓ with N_B turns (Fig. 32.15b), carrying a current I, and having a cross-sectional area A. The handle coil contains N_H turns and completely surrounds the base coil. Find the mutual inductance of the system.



$$B = \frac{\mu_0 N_B I}{\ell}$$

$$M = \frac{N_H \Phi_{BH}}{I} = \frac{N_H B A}{I} = \mu_0 \frac{N_B N_H A}{\ell}$$

Oscillations in an *LC* Circuit



$$U = U_C + U_L = \frac{Q^2}{2C} + \tfrac{1}{2}LI^2$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C} + \tfrac{1}{2}LI^2\right) = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

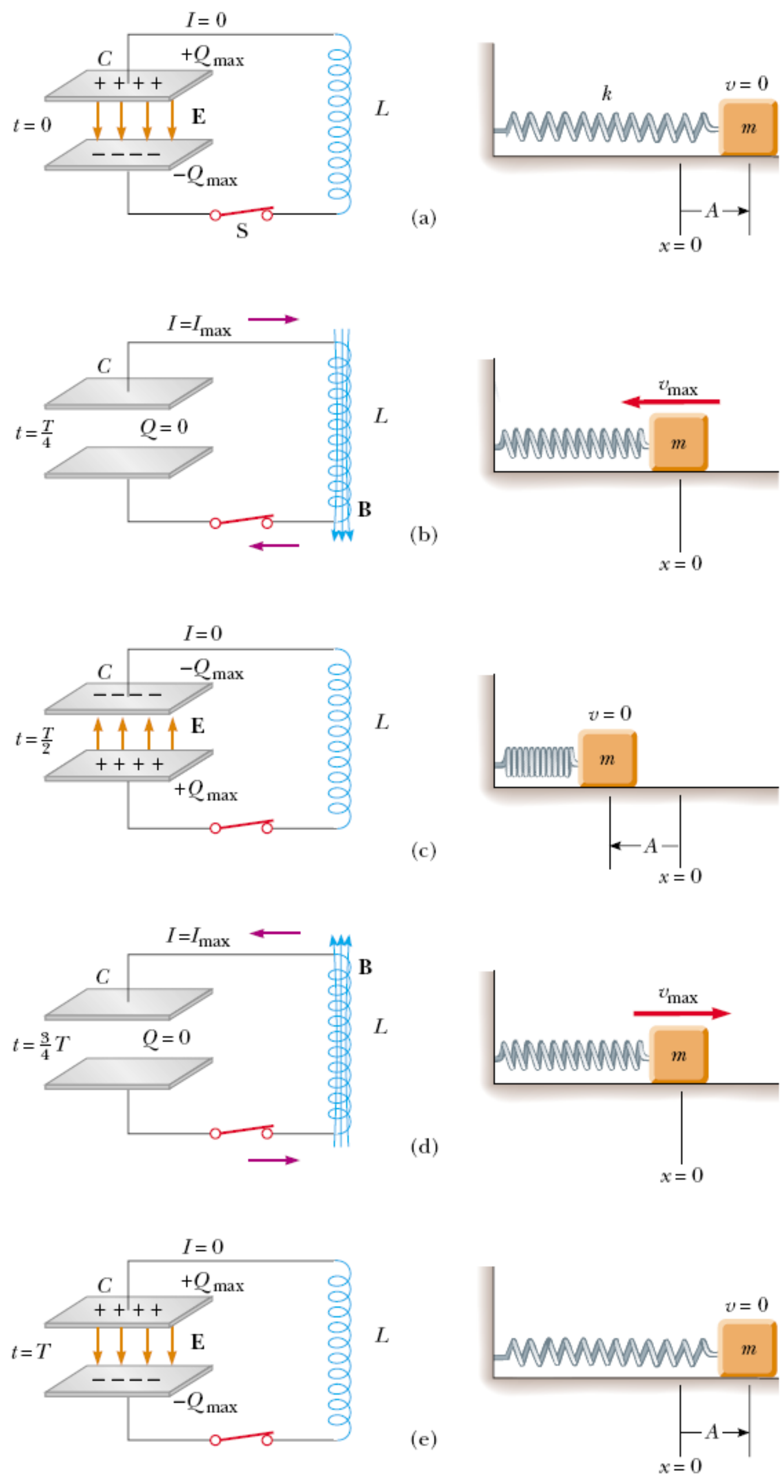
$$\omega = \sqrt{k/m}$$

$$x = A\cos(\omega t + \phi)$$

$$Q = Q_{max}\cos(\omega t + \phi)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$I = \frac{dQ}{dt} = -\omega Q_{max}\sin(\omega t + \phi)$$



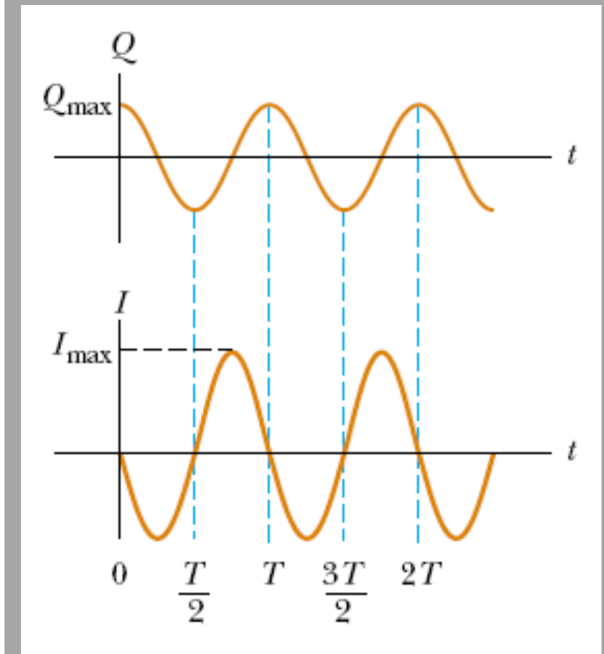
To determine the value of the phase angle ϕ, we examine the initial conditions, which in our situation require that at $t = 0$, $I = 0$ and $Q = Q_{max}$. Setting $I = 0$ at $t = 0$ in Equation 32.23, we have

$$0 = -\omega Q_{max} \sin \phi$$

$$Q = Q_{max} \cos \omega t$$

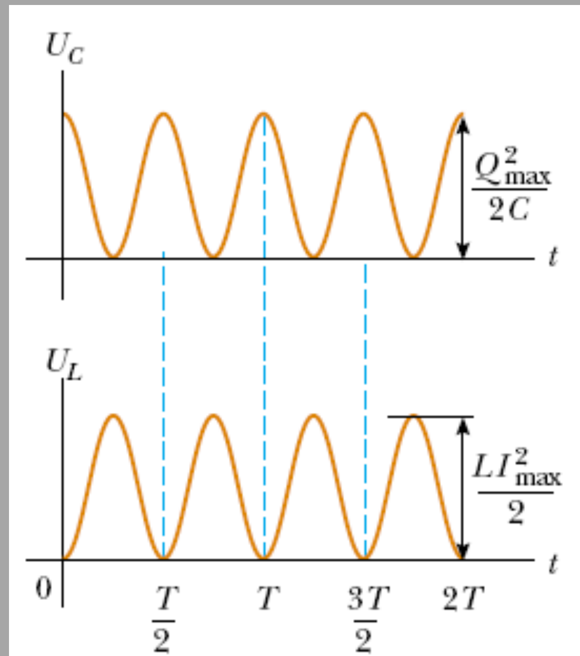
$$I = -\omega Q_{max} \sin \omega t = -I_{max} \sin \omega t$$

$$U = U_C + U_L = \frac{Q_{max}^2}{2C} \cos^2 \omega t + \tfrac{1}{2} L I_{max}^2 \sin^2 \omega t$$

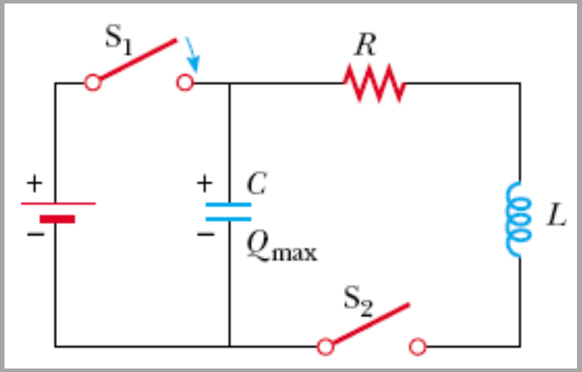


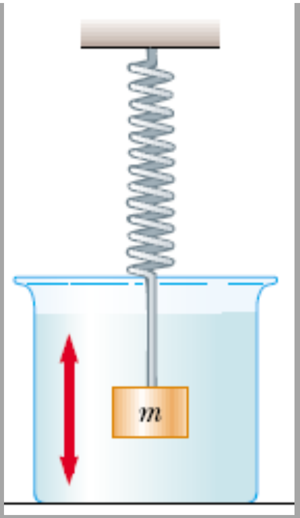
$$\frac{Q_{max}^2}{2C} = \frac{L I_{max}^2}{2}$$

$$U = \frac{Q_{max}^2}{2C} (\cos^2 \omega t + \sin^2 \omega t) = \frac{Q_{max}^2}{2C}$$



The *RLC* Circuit





$$\frac{dU}{dt} = -I^2R$$

$$LI\frac{dI}{dt} + \frac{Q}{C}\frac{dQ}{dt} = -I^2R$$

$$LI\frac{d^2Q}{dt^2} + I^2R + \frac{Q}{C}I = 0$$

$$L\frac{d^2Q}{dt^2} + IR + \frac{Q}{C} = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

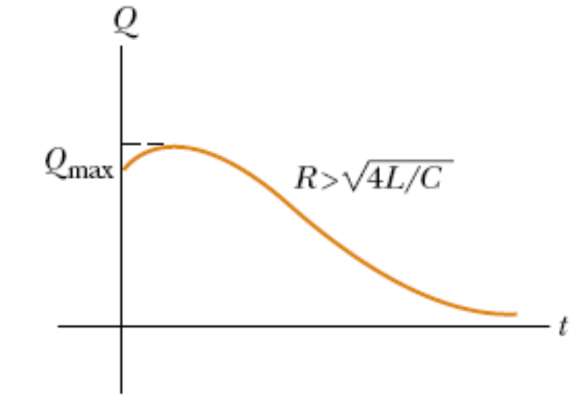
Analogies Between Electrical and Mechanical Systems

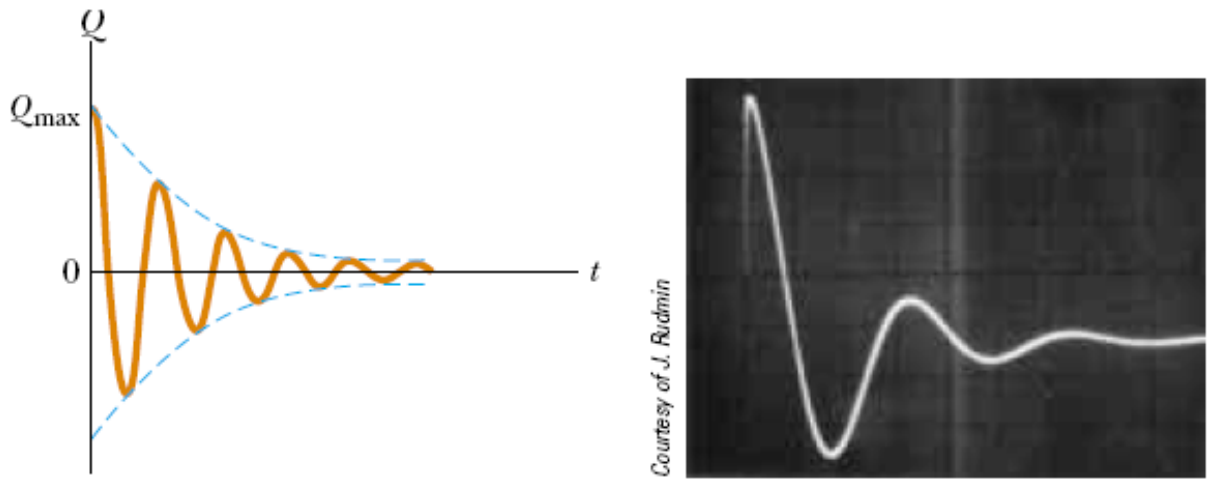
Electric Circuit		One-Dimensional Mechanical System
Charge	$Q \leftrightarrow x$	Position
Current	$I \leftrightarrow v_x$	Velocity
Potential difference	$\Delta V \leftrightarrow F_x$	Force
Resistance	$R \leftrightarrow b$	Viscous damping coefficient
Capacitance	$C \leftrightarrow 1/k$	(k = spring constant)
Inductance	$L \leftrightarrow m$	Mass
Current = time derivative of charge	$I = \frac{dQ}{dt} \leftrightarrow v_x = \frac{dx}{dt}$	Velocity = time derivative of position
Rate of change of current = second time derivative of charge	$\frac{dI}{dt} = \frac{d^2Q}{dt^2} \leftrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	Acceleration = second time derivative of position
Energy in inductor	$U_L = \frac{1}{2}LI^2 \leftrightarrow K = \frac{1}{2}mv^2$	Kinetic energy of moving object
Energy in capacitor	$U_C = \frac{1}{2}\frac{Q^2}{C} \leftrightarrow U = \frac{1}{2}kx^2$	Potential energy stored in a spring
Rate of energy loss due to resistance	$I^2R \leftrightarrow bv^2$	Rate of energy loss due to friction
RLC circuit	$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \leftrightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$	Damped object on a spring

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

$$Q = Q_{\max} e^{-Rt/2L} \cos \omega_d t$$

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$$





(a) (b)

Active Figure 32.23 (a) Charge versus time for a damped *RLC* circuit. The charge decays in this way when $R > \sqrt{4L/C}$. The Q-versus-t curve represents a plot of Equation 32.31. (b) Oscilloscope pattern showing the decay in the oscillations of an *RLC* circuit.