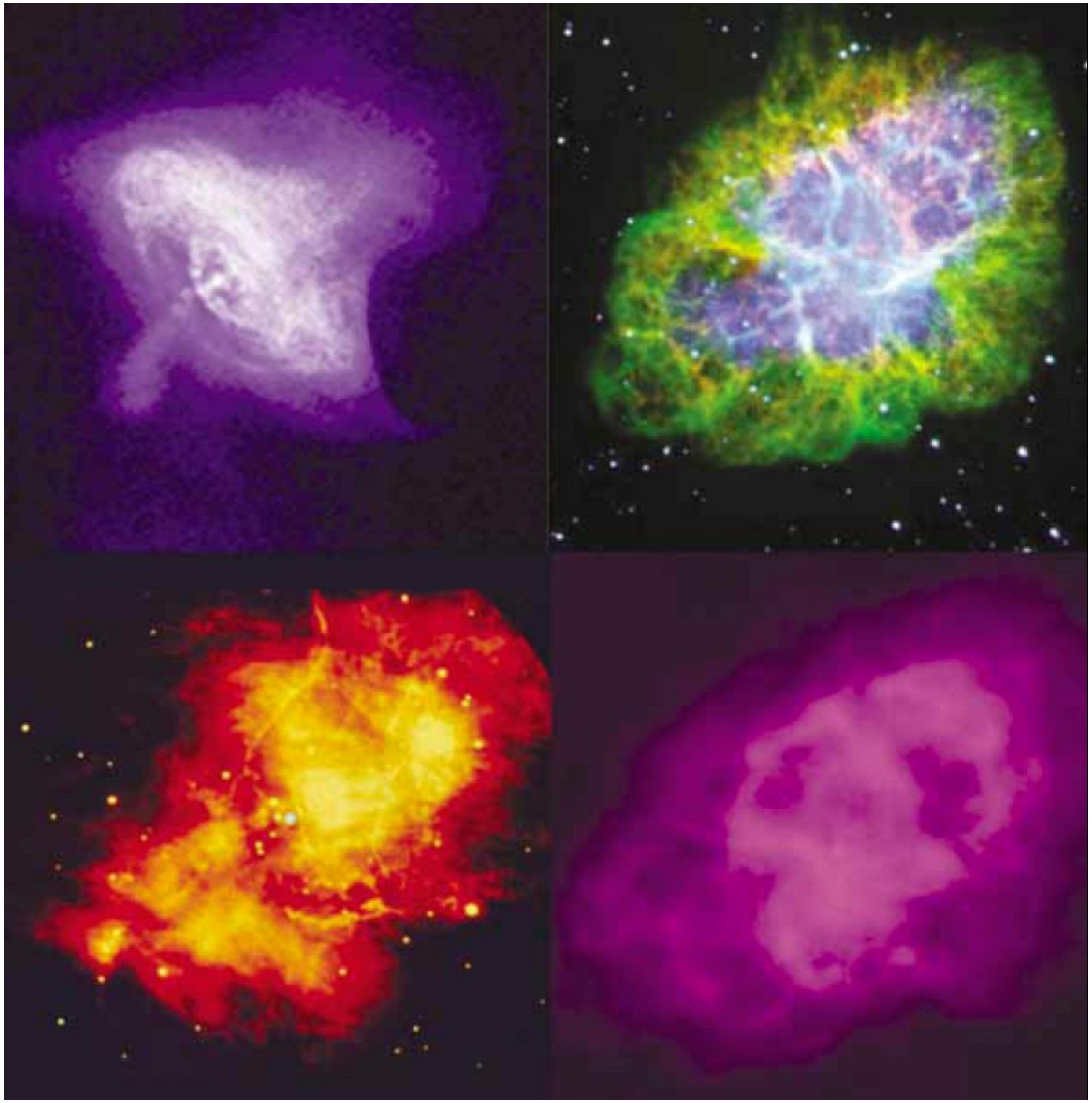
Electromagnetic Waves



Maxwell's Equations and Hertz's Discoveries

Plane Electromagnetic Waves

Energy Carried by Electromagnetic Waves

Momentum and Radiation Pressure

Production of Electromagnetic Waves by an Antenna

The Spectrum of Electromagnetic Waves

Maxwell's Equations and Hertz's Discoveries

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

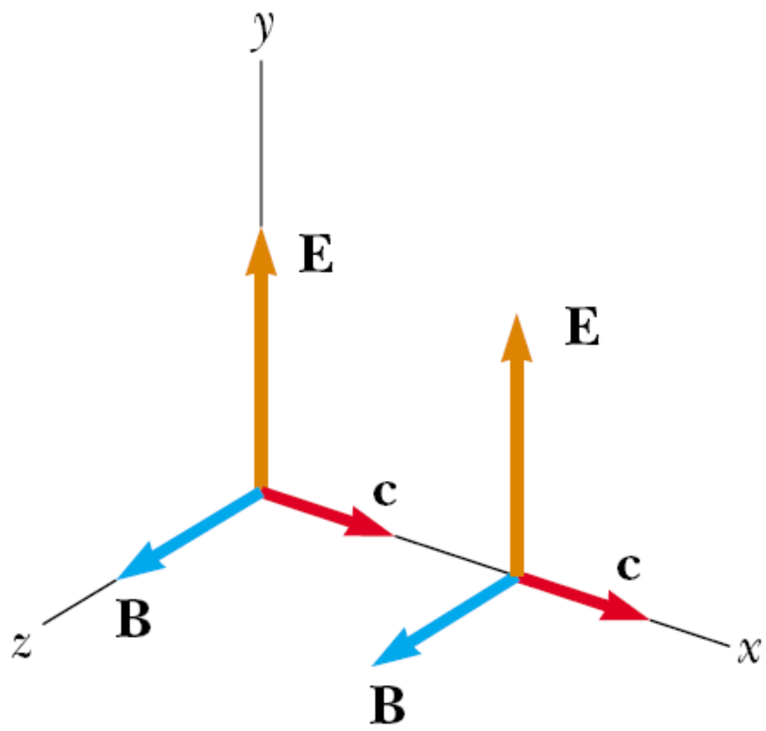
Plane Electromagnetic Waves

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$



$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) = -\frac{\partial}{\partial t}\left(-\mu_0\epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

$$E = E_{\max} \cos(kx - \omega t)$$

$$B = B_{\max} \cos(kx - \omega t)$$

$$\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 B}{\partial t^2}}$$

$$\boxed{c = \frac{1}{\sqrt{\mu_0\epsilon_0}}}$$

$$E = E_{\text{max}} \cos(kx - \omega t)$$

$$B = B_{\text{max}} \cos(kx - \omega t)$$

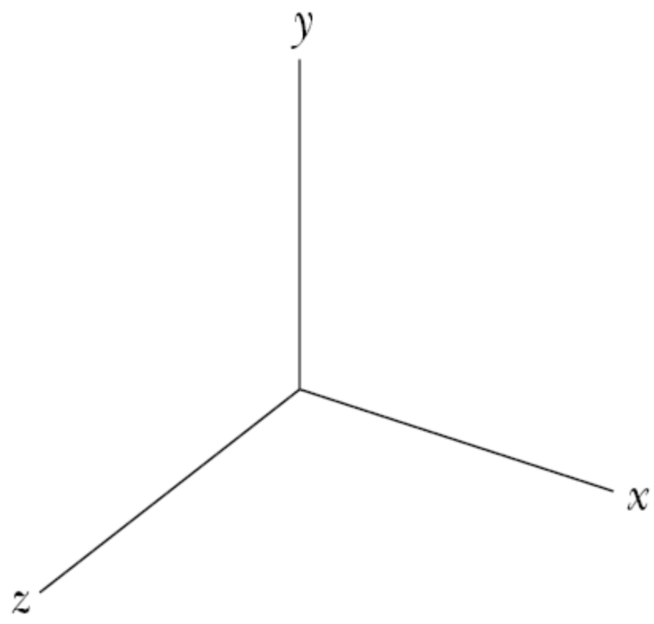
$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f = c$$

$$v = c = \lambda f,$$

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{f}$$

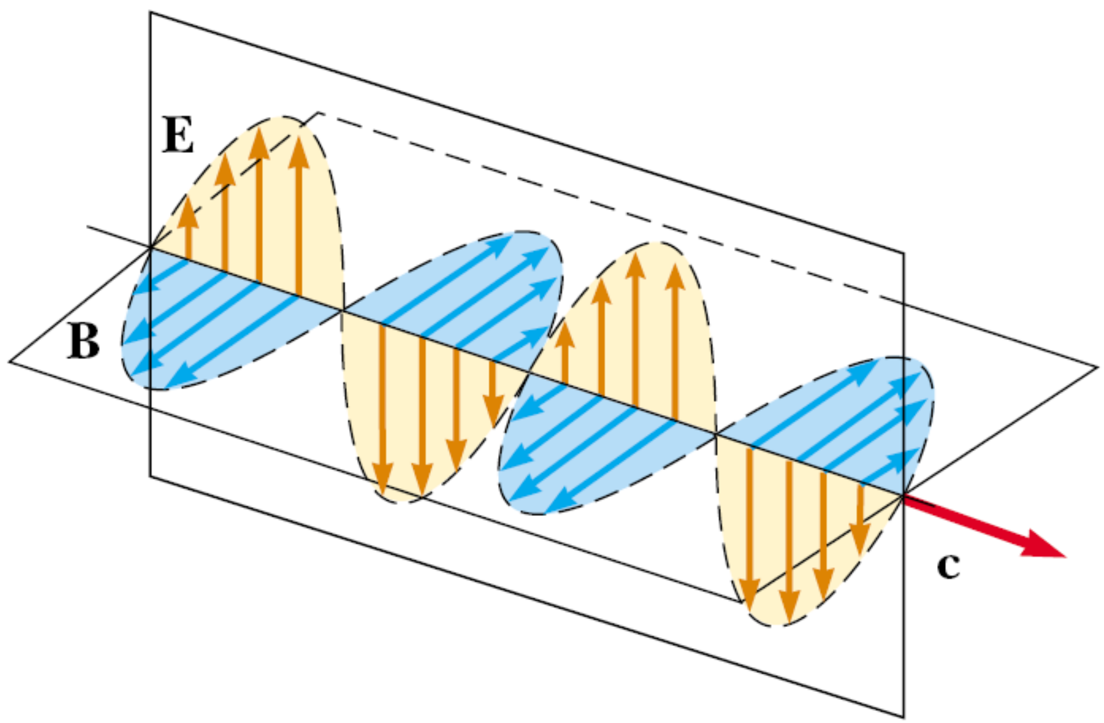
$$\frac{\partial E}{\partial x} = -kE_{\text{max}} \sin(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = \omega B_{\text{max}} \sin(kx - \omega t)$$



$$E = E_{\max} \cos(kx - \omega t)$$

$$B = B_{\max} \cos(kx - \omega t)$$



$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$E = E_{\max} \cos(kx - \omega t)$$

$$B = B_{\max} \cos(kx - \omega t)$$

$$kE_{\max} = \omega B_{\max}$$

$$\frac{E_{\max}}{B_{\max}} = \frac{\omega}{k} = c$$

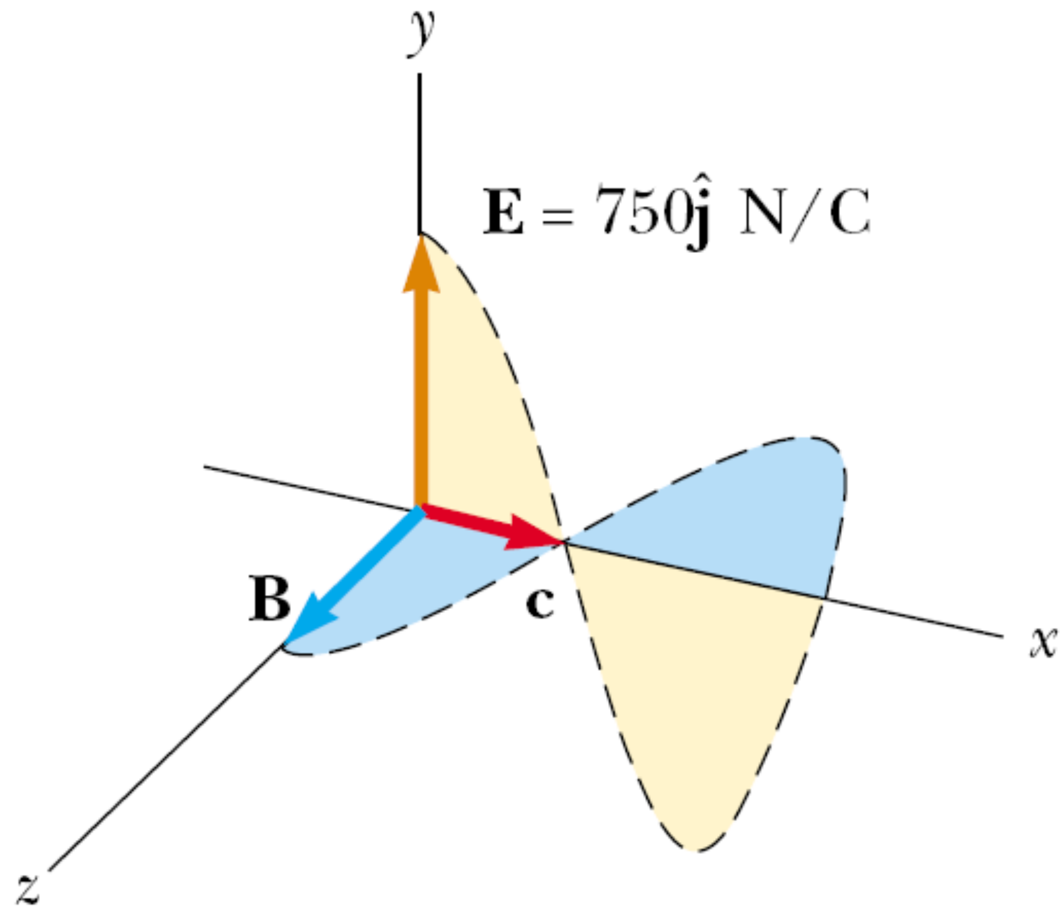
$$\boxed{\frac{E_{\max}}{B_{\max}} = \frac{E}{B} = c}$$

- The magnitudes of **E** and **B** in empty space are related by the expression $E/B = c$.
- Electromagnetic waves obey the principle of superposition.

Example 34.1 An Electromagnetic Wave

A sinusoidal electromagnetic wave of frequency 40.0 MHz travels in free space in the x direction, as in Figure 34.4.

(A) Determine the wavelength and period of the wave.



$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{4.00 \times 10^7 \text{ s}^{-1}} = \boxed{7.50 \text{ m}}$$

$$T = \frac{1}{f} = \frac{1}{4.00 \times 10^7 \text{ s}^{-1}} = \boxed{2.50 \times 10^{-8} \text{ s}}$$

(B) At some point and at some instant, the electric field has its maximum value of 750 N/C and is along the y axis. Calculate the magnitude and direction of the magnetic field at this position and time.

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = \frac{750 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.50 \times 10^{-6} \text{ T}}$$

(C) Write expressions for the space–time variation of the components of the electric and magnetic fields for this wave.

$$E = E_{\max} \cos(kx - \omega t) = (750 \text{ N/C}) \cos(kx - \omega t)$$

$$B = B_{\max} \cos(kx - \omega t) = (2.50 \times 10^{-6} \text{ T}) \cos(kx - \omega t)$$

where

$$\omega = 2\pi f = 2\pi(4.00 \times 10^{7} \text{ s}^{-1}) = 2.51 \times 10^{8} \text{ rad/s}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.50 \text{ m}} = 0.838 \text{ rad/m}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

Derivation of Equations

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$E(x + dx, t) \approx E(x, t) + \left.\frac{dE}{dx}\right]_{t \text{ constant}} dx = E(x, t) + \frac{\partial E}{\partial x} dx$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = [E(x + dx, t)]\ell - [E(x, t)]\ell \approx \ell \left(\frac{\partial E}{\partial x}\right) dx$$

$$\frac{d\Phi_B}{dt} = \ell\, dx \left.\frac{dB}{dt}\right]_{x \text{ constant}} = \ell\, dx \frac{\partial B}{\partial t}$$

$$\ell \left(\frac{\partial E}{\partial x}\right) dx = -\ell\, dx \frac{\partial B}{\partial t}$$

$$\boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = [B(x, t)]\ell - [B(x + dx, t)]\ell \approx -\ell \left(\frac{\partial B}{\partial x}\right) dx$$

$$\frac{\partial \Phi_E}{\partial t} = \ell\, dx \frac{\partial E}{\partial t}$$

$$-\ell \left(\frac{\partial B}{\partial x}\right) dx = \mu_0 \epsilon_0 \ell\, dx \left(\frac{\partial E}{\partial t}\right)$$

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

Energy Carried by Electromagnetic Waves

Poynting vector,

$$\boxed{\mathbf{S} \equiv \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}}$$

$$S = \frac{EB}{\mu_0}$$

$$S = \frac{E^2}{\mu_0 c} = \frac{c}{\mu_0} B^2$$

$$\boxed{I = S_{\text{av}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_{\max}^2}$$

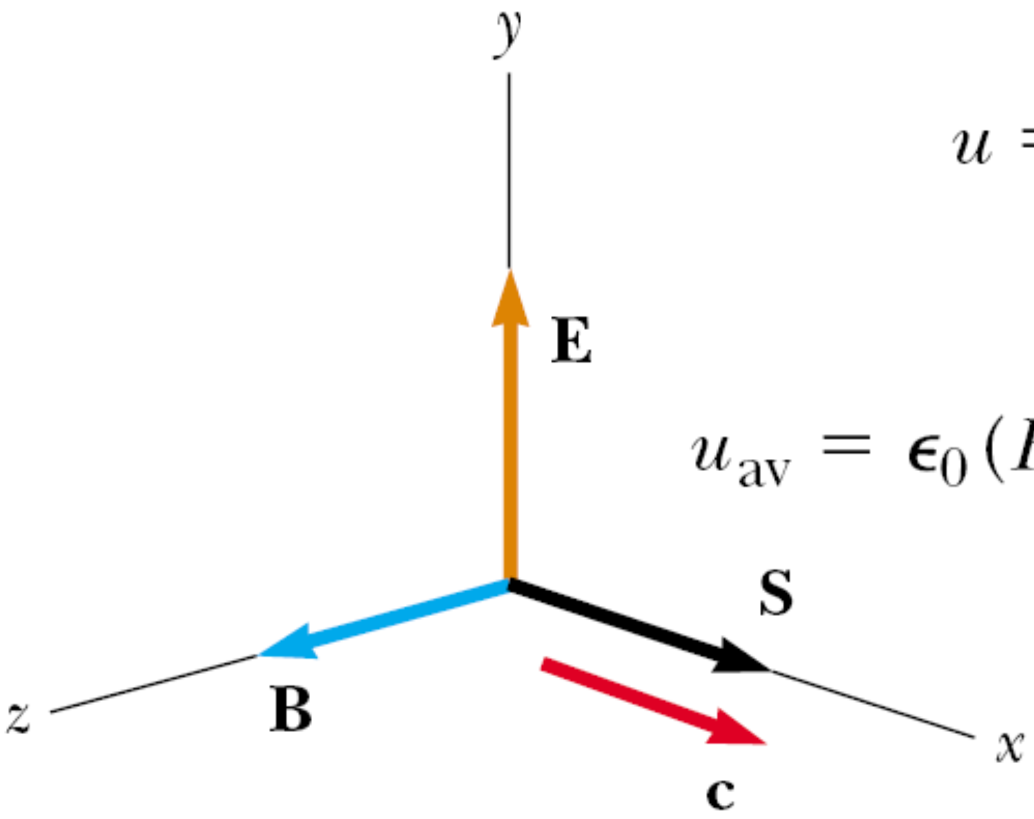
$$u_E = \frac{1}{2}\epsilon_0 E^2$$

$$u_B = \frac{B^2}{2\mu_0}$$

$$u_B = \frac{(E/c)^2}{2\mu_0} = \frac{\epsilon_0\mu_0}{2\mu_0}E^2 = \frac{1}{2}\epsilon_0 E^2$$

$$u_B = u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

$$u = u_E + u_B = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$$



$$u_{av} = \epsilon_0 (E^2)_{av} = \frac{1}{2}\epsilon_0 E_{max}^2 = \frac{B_{max}^2}{2\mu_0}$$

$$I = S_{av} = c\,u_{av}$$

Momentum and Radiation Pressure

$$p = \frac{U}{c} \qquad \text{(complete absorption)}$$

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt}$$

$$P = \frac{1}{A}\frac{dp}{dt} = \frac{1}{A}\frac{d}{dt}\left(\frac{U}{c}\right) = \frac{1}{c}\frac{(dU/dt)}{A}$$

$$P = \frac{S}{c}$$

$$p = \frac{2U}{c} \qquad \text{(complete reflection)}$$

$$P = \frac{2S}{c}$$

The Spectrum of Electromagnetic Waves