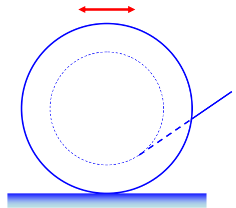
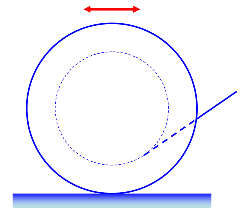
Example: Giant Yo-Yo

- When one pulls on rope, does spool move right or left?



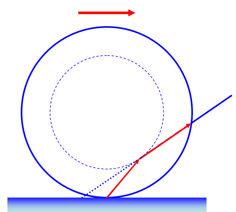
Example: Giant Yo-Yo

- When one pulls on rope, does spool move right or left?
  - Depends on torque about pivot point (contact with floor)



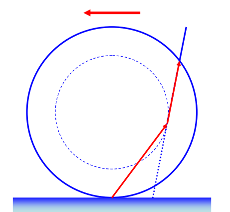
Example: Giant Yo-Yo

- When one pulls on rope, does spool move right or left?
  - Depends on torque about pivot point (contact with floor)
  - If rope projects to a point behind pivot, rolls right



Example: Giant Yo-Yo

- When one pulls on rope, does spool move right or left?
  - Depends on torque about pivot point (contact with floor)
  - If rope projects to a point behind pivot, rolls right
  - If rope projects in front, rolls left



Torque as a Vector

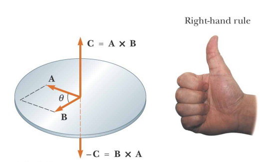
- Torque is a vector
  - Magnitude

    $\tau = rF\sin\phi$
  - What is direction?
  - Define torque vector with cross product of position and force vectors

    $\boldsymbol{\tau} \equiv \mathbf{r} \times \mathbf{F}$

The Vector Cross Product

- If **C** = **A** x **B**, then
  1) $C = AB\sin\theta$
  2) Direction of **C** given by Right Hand Rule
- Properties



$\mathbf{A}\times\mathbf{B} = -\mathbf{B}\times\mathbf{A}$

Parallel $\mathbf{A}$ and $\mathbf{B}$: $\mathbf{A}\times\mathbf{B} = 0$, $\mathbf{A}\times\mathbf{A} = 0$

Perpendicular $\mathbf{A}$ and $\mathbf{B}$: $|\mathbf{A}\times\mathbf{B}| = AB$

$\mathbf{A}\times(\mathbf{B}+\mathbf{C}) = \mathbf{A}\times\mathbf{B} + \mathbf{A}\times\mathbf{C}$

$\frac{d}{dt}(\mathbf{A}\times\mathbf{B}) = \frac{d\mathbf{A}}{dt}\times\mathbf{B} + \mathbf{A}\times\frac{d\mathbf{B}}{dt}$

The Vector Cross Product

- Unit Vectors and the algebraic form of cross product

$$\hat{\mathbf{i}}\times\hat{\mathbf{j}}=\hat{\mathbf{k}}$$
$$\hat{\mathbf{j}}\times\hat{\mathbf{k}}=\hat{\mathbf{i}}$$
$$\hat{\mathbf{k}}\times\hat{\mathbf{i}}=\hat{\mathbf{j}}$$

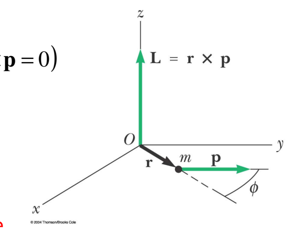
$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$
$$=\begin{vmatrix}A_y & A_z\\ B_y & B_z\end{vmatrix}\hat{\mathbf{i}}+\begin{vmatrix}A_z & A_x\\ B_z & B_x\end{vmatrix}\hat{\mathbf{j}}+\begin{vmatrix}A_x & A_y\\ B_x & B_y\end{vmatrix}\hat{\mathbf{k}}$$
$$=\left(A_yB_z-A_zB_y\right)\hat{\mathbf{i}}+\left(A_zB_x-A_xB_z\right)\hat{\mathbf{j}}+\left(A_xB_y-A_yB_x\right)\hat{\mathbf{k}}$$

Angular Momentum

- Consider the torque on a particle

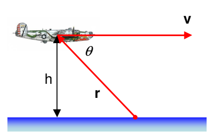
$$\boldsymbol{\tau}=\mathbf{r}\times\mathbf{F}=\mathbf{r}\times\tfrac{d\mathbf{p}}{dt}$$
$$=\frac{d\left(\mathbf{r}\times\mathbf{p}\right)}{dt}\quad\left(\text{since }\tfrac{d\mathbf{r}}{dt}\times\mathbf{p}=0\right)$$
$$=\frac{d\mathbf{L}}{dt}$$

- Call **L** = **r** x **p** the angular momentum
  - (of the particle about the origin)



Example: Airplane Overhead

- What is the angular momentum of a plane flying overhead?
  - It's not zero!
  - Even if plane is flying level



$$\mathbf{L}=\mathbf{r}\times\mathbf{p}$$
$$=\mathbf{r}\times m\mathbf{v}$$
$$\left|\mathbf{L}\right|=mvr\sin\theta=mvh$$

  - Note that **L** has a constant magnitude with a direction into the page

Angular Momentum

- Now consider a system of particles

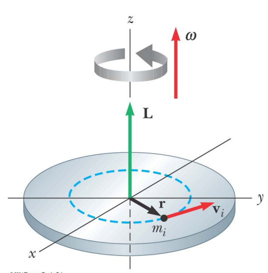
$$\mathbf{r}\times\sum\mathbf{F}_{\text{ext}}=\mathbf{r}\times\frac{d\mathbf{P}_{\text{tot}}}{dt}$$
$$\sum\left(\mathbf{r}\times\mathbf{F}_{\text{ext}}\right)=\frac{d\left(\mathbf{r}\times\mathbf{P}_{\text{tot}}\right)}{dt}$$
$$\sum\boldsymbol{\tau}_{\text{ext}}=\sum_i\boldsymbol{\tau}_i=\sum_i\frac{d\mathbf{L}_i}{dt}=\frac{d\mathbf{L}_{\text{tot}}}{dt}$$

- Like force and momentum, the net torque gives the rate of change of angular momentum

Spinning Rigid Objects

- What's the angular momentum of a spinning, rigid object?
  - Always measure from (and perpendicular to) axis of rotation (which is along z axis)

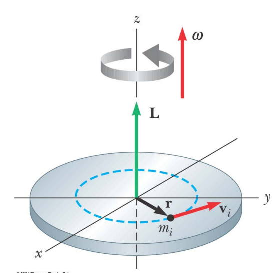
$$\mathbf{L}=L_z\hat{\mathbf{k}}=\sum_i\left(\mathbf{r}_i\times\mathbf{p}_i\right)=\sum_i r_ip_i\hat{\mathbf{k}}$$
$$L_z=\sum_i m_ir_iv_i$$
$$=\sum_i m_ir_i^2\omega$$
$$=I\omega$$



Spinning Rigid Objects

- What happens when we apply a torque to a spinning object?

$$L_z=I\omega$$
$$\sum\left(\tau_{\text{ext}}\right)_z=\frac{dL_z}{dt}$$
$$=I\frac{d\omega}{dt}$$
$$=I\alpha$$



Translational-Angular Analogs

$$x \Leftrightarrow \theta$$
$$\mathbf{v} \Leftrightarrow \boldsymbol{\omega}$$
$$\mathbf{a} \Leftrightarrow \boldsymbol{\alpha}$$
$$m \Leftrightarrow I$$
$$\mathbf{F} \Leftrightarrow \boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$
$$\mathbf{p} = m\mathbf{v} \Leftrightarrow \mathbf{L} = \mathbf{r} \times \mathbf{P}$$
$$K = \tfrac{1}{2} m v^2 \Leftrightarrow K = \tfrac{1}{2} I \omega^2$$
$$\sum \mathbf{F} = m\mathbf{a} = \frac{d\mathbf{p}}{dt} \Leftrightarrow \sum \boldsymbol{\tau} = I\boldsymbol{\alpha} = \frac{d\mathbf{L}}{dt}$$

Conservation of Angular Momentum

- If there are no external torques, angular momentum cannot change!

$$\sum \boldsymbol{\tau} = \frac{d\mathbf{L}}{dt} = 0 \Rightarrow \mathbf{L} = \text{const.}$$

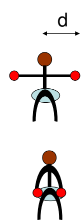
  - Like linear momentum, angular momentum is *conserved*
  - If moment-of-inertia changes, so does angular velocity!

$$I_i \omega_i = I_f \omega_f$$

Example: Person on Stool

- If a person is spinning on a stool holding weights, they spin faster if they pull them in

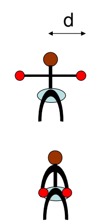
$$I_i = I_{\text{person}} + 2md^2$$
$$I_f = I_{\text{person}}$$
$$L_i = I_i \omega_i = L_f = I_f \omega_f$$
$$\omega_f = \frac{I_{\text{person}} + 2md}{I_{\text{person}}} \omega_i$$



Example: Person on Stool

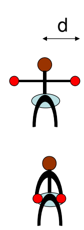
- **L** is conserved, is kinetic energy?

$$K_f = \tfrac{1}{2} I_f \omega_f^2$$
$$= \tfrac{1}{2} I_f \left(\frac{I_i}{I_f} \omega_i \right)^2$$
$$= \frac{I_i}{I_f} K_i$$
$$> K_i$$



Example: Person on Stool

- Where did energy come from?
  - Work required to pull in weights!



Example: P with Wheel on Stool

- What happens when the wheel in the picture is turned over?

$$L_i = L_{\text{wheel}}$$
$$L_f = L_{\text{person}} - L_{\text{wheel}} = L_i$$
$$L_{\text{person}} = 2L_{\text{wheel}}$$

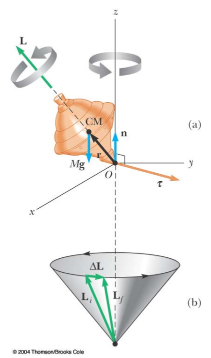
  - Since **L** is conserved, but wheel's **L** changes sign, the person on the stool must start spinning to make up the difference!
    - But which way?

Gyroscopes and Tops

- Torque is a vector!

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \frac{d\mathbf{L}}{dt}$$

- Examples (pulleys, etc) have had $\boldsymbol{\tau}$ and **L** parallel
- What if they're not parallel?
  - What if **r** and **L** are parallel? Then we have a gyroscope or a top.



Gyroscopes and Tops

- Find $\boldsymbol{\tau}$ and **L** for top

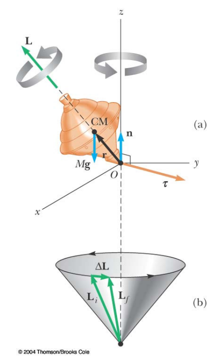
$$\mathbf{L} = I\boldsymbol{\omega} = (I\omega\cos\theta)\hat{\mathbf{i}} + (I\omega\sin\theta)\hat{\mathbf{k}}$$

$$\boldsymbol{\tau} = \mathbf{r}_{\mathrm{CM}} \times \mathbf{F} = (mgr_{\mathrm{CM}}\sin\theta)\hat{\mathbf{j}}$$

- Now, what is $d\mathbf{L}$?

$$d\mathbf{L} = \boldsymbol{\tau}dt = (mgr_{\mathrm{CM}}\sin\theta \cdot dt)\hat{\mathbf{j}}$$

- So the **L** vector moves perpendicular to itself.
  - It doesn't change magnitude, so it must sweep out a circle
  - Call this precession

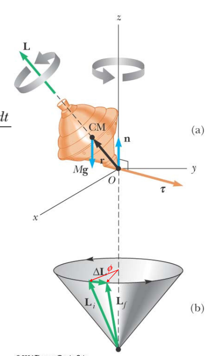


Gyroscopes and Tops

- How fast is the precession?

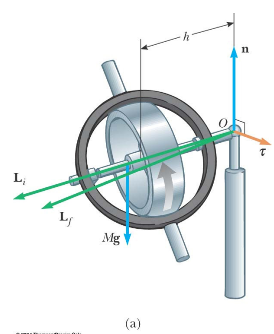
$$\sin(d\phi) \approx d\phi = \frac{dL}{L} = \frac{\tau\, dt}{L} = \frac{(Mgh)\, dt}{L}$$

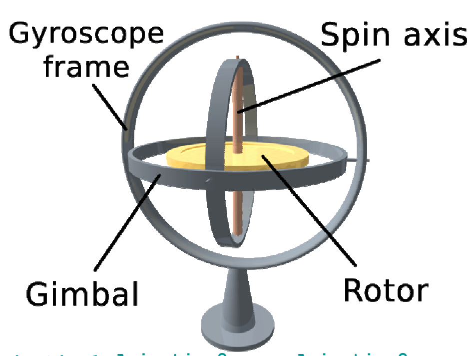
$$\omega_p = \frac{d\phi}{dt} = \frac{Mgh}{I\omega}$$



Gyroscopes and Tops

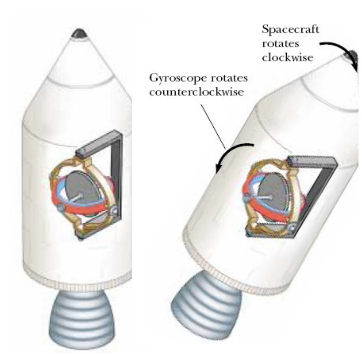
- Consider spinning gyroscope *held* horizontally
  - **L** is horizontal
- When end is dropped will begin to precess, giving **L** a vertical component
  - Where does this component come from? There's no vertical $\boldsymbol{\tau}$!
  - In fact end will bob up and down so that there is never any vertical **L**





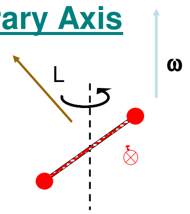
Animation1 Animation2 Animation2

As an example of the usefulness of gyroscopes, suppose you are in a spacecraft in deep space and you need to alter your trajectory. You need to turn the spacecraft around in order to fire the engines in the correct direction. But how do you turn a spacecraft around in empty space?



Rotation about Arbitrary Axis

- Consider rotating stick with masses
  - To keep it spinning about this axis, a torque must be applied
  - Thus **L** not constant, but spins around
  - What about **L**=*I*$\boldsymbol{\omega}$?
  - In fact **I** is a tensor (like a vector, but with two spatial indices).



$$\mathbf{L} = \ddot{\mathbf{I}}\boldsymbol{\omega}$$

$$\begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

[1] In general, the expression $\mathbf{L} = I\boldsymbol{\omega}$ is not always valid. If a rigid object rotates about an *arbitrary* axis, $\mathbf{L}$ and $\boldsymbol{\omega}$ may point in different directions. In this case, the moment of inertia cannot be treated as a scalar. Strictly speaking, $\mathbf{L} = I\boldsymbol{\omega}$ applies only to rigid objects of any shape that rotate about one of three mutually perpendicular axes (called *principal axes*) through the center of mass. This is discussed in more advanced texts on mechanics.

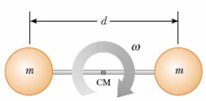


Figure 11.17 The rigid-rotor model of a diatomic molecule. The rotation occurs about the center of mass in the plane of the page.

$$\hbar = h/2\pi$$

$$\hbar = 1.054 \times 10^{-34}\ \mathrm{kg \cdot m^2/s}$$

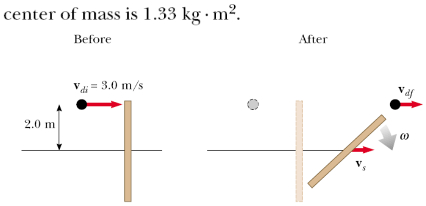
$$I_{\mathrm{CM}}\omega \approx \hbar \qquad \text{or} \qquad \omega \approx \frac{\hbar}{I_{\mathrm{CM}}}$$

$$I_{CM}(O_2) = 1.95 \times 10^{-46}\ kgm^2$$

$$\omega \approx \frac{\hbar}{I_{\mathrm{CM}}} = \frac{1.054 \times 10^{-34}\ \mathrm{kg \cdot m^2/s}}{1.95 \times 10^{-46}\ \mathrm{kg \cdot m^2}} \sim 10^{12}\ \mathrm{rad/s}$$

Example 11.10 Disk and Stick

A 2.0-kg disk traveling at 3.0 m/s strikes a 1.0-kg stick of length 4.0 m that is lying flat on nearly frictionless ice, as shown in Figure 11.13. Assume that the collision is elastic and that the disk does not deviate from its original line of motion. Find the translational speed of the disk, the translational speed of the stick, and the angular speed of the stick after the collision. The moment of inertia of the stick about its center of mass is 1.33 kg · m^2.



$$p_i = p_f$$

$$m_d v_{di} = m_d v_{df} + m_s v_s$$

$$(2.0\ \mathrm{kg})(3.0\ \mathrm{m/s}) = (2.0\ \mathrm{kg})v_{df} + (1.0\ \mathrm{kg})v_s$$

$$6.0\ \mathrm{kg \cdot m/s} - (2.0\ \mathrm{kg})v_{df} = (1.0\ \mathrm{kg})v_s$$

$$L_i = L_f$$

$$-rm_d v_{di} = -rm_d v_{df} + I\omega$$

$$-(2.0\ \mathrm{m})(2.0\ \mathrm{kg})(3.0\ \mathrm{m/s}) = -(2.0\ \mathrm{m})(2.0\ \mathrm{kg})v_{df} + (1.33\ \mathrm{kg \cdot m^2})\omega$$

$$-12\ \mathrm{kg \cdot m^2/s} = -(4.0\ \mathrm{kg \cdot m})v_{df} + (1.33\ \mathrm{kg \cdot m^2})\omega$$

$$-9.0\ \mathrm{rad/s} + (3.0\ \mathrm{rad/m})v_{df} = \omega$$

$$K_i = K_f$$

$$\tfrac{1}{2}m_d v_{di}^2 = \tfrac{1}{2}m_d v_{df}^2 + \tfrac{1}{2}m_s v_s^2 + \tfrac{1}{2}I\omega^2$$

$$\tfrac{1}{2}(2.0\ \mathrm{kg})(3.0\ \mathrm{m/s})^2 = \tfrac{1}{2}(2.0\ \mathrm{kg})v_{df}^2 + \tfrac{1}{2}(1.0\ \mathrm{kg})v_s^2 + \tfrac{1}{2}(1.33\ \mathrm{kg \cdot m^2})\omega^2$$

$$18\ \mathrm{m^2/s^2} = 2.0\ v_{df}^2 + v_s^2 + (1.33\ \mathrm{m^2})\omega^2$$

In solving Equations (1), (2), and (3) simultaneously, we find that v_{df} = 2.3 m/s, v_s = 1.3 m/s, and ω = −2.0 rad/s.

What If? What if the collision between the disk and the stick is perfectly inelastic? How does this change the analysis?

$$p_i = p_f$$

$$m_d v_{di} = (m_d + m_s) v_{CM}$$

$$(2.0\text{ kg})(3.0\text{ m/s}) = (2.0\text{ kg} + 1.0\text{ kg}) v_{CM}$$

$$v_{CM} = 2.0\text{ m/s}$$

$$y_{CM} = \frac{(2.0\text{ kg})(2.0\text{ m}) + (1.0\text{ kg})(0)}{(2.0\text{ kg} + 1.0\text{ kg})} = 1.33\text{ m}$$

$$L_i = L_f$$

$$-r m_d v_{di} = I_d \omega + I_s \omega$$

$$-(0.67\text{ m}) m_d v_{di} = [m_d (0.67\text{ m})^2]\omega + I_s \omega$$

$$I_s = I_{CM} + MD^2$$
$$= 1.33\text{ kg}\cdot\text{m}^2 + (1.0\text{ kg})(1.33\text{ m})^2 = 3.1\text{ kg}\cdot\text{m}^2$$

$$-(0.67\text{ m})(2.0\text{ kg})(3.0\text{ m/s}) = [(2.0\text{ kg})(0.67\text{ m})^2]\omega + (3.1\text{ kg}\cdot\text{m}^2)\omega$$

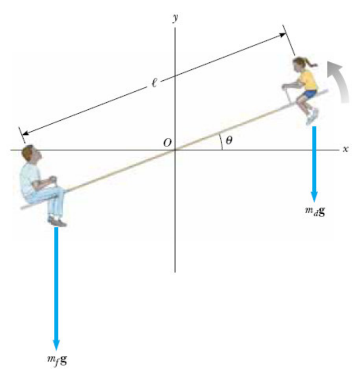
$$-4.0\text{ kg}\cdot\text{m}^2/\text{s} = (4.0\text{ kg}\cdot\text{m}^2)\omega$$

$$\omega = \frac{-4.0\text{ kg}\cdot\text{m}^2/\text{s}}{4.0\text{ kg}\cdot\text{m}^2} = -1.0\text{ rad/s}$$

Example 11.6 The Seesaw

A father of mass m_f and his daughter of mass m_d sit on opposite ends of a seesaw at equal distances from the pivot at the center (Fig. 11.9). The seesaw is modeled as a rigid rod of mass M and length ℓ and is pivoted without friction. At a given moment, the combination rotates in a vertical plane with an angular speed ω.

(A) Find an expression for the magnitude of the system's angular momentum.



Solution The moment of inertia of the system equals the sum of the moments of inertia of the three components: the seesaw and the two individuals, whom we will model as particles. Referring to Table 10.2 to obtain the expression for the moment of inertia of the rod, and using the expression $I = mr^2$ for each person, we find that the total moment of inertia about the z axis through O is

$$I = \tfrac{1}{12}M\ell^2 + m_f\left(\frac{\ell}{2}\right)^2 + m_d\left(\frac{\ell}{2}\right)^2 = \frac{\ell^2}{4}\left(\frac{M}{3} + m_f + m_d\right)$$

Therefore, the magnitude of the angular momentum is

$$L = I\omega = \frac{\ell^2}{4}\left(\frac{M}{3} + m_f + m_d\right)\omega$$

(B) Find an expression for the magnitude of the angular acceleration of the system when the seesaw makes an angle θ with the horizontal.

Solution To find the angular acceleration of the system at any angle θ, we first calculate the net torque on the system and then use $\sum \tau_{ext} = I\alpha$ to obtain an expression for α.

The torque due to the force $m_f g$ about the pivot is

$$\tau_f = m_f g \frac{\ell}{2}\cos\theta \qquad (\boldsymbol{\tau}_f \text{ out of page})$$

The torque due to the force $m_d g$ about the pivot is

$$\tau_d = -m_d g \frac{\ell}{2}\cos\theta \qquad (\boldsymbol{\tau}_d \text{ into page})$$

Hence, the net torque exerted on the system about O is

$$\sum \tau_{\text{ext}} = \tau_f + \tau_d = \tfrac{1}{2}(m_f - m_d)g\ell \cos\theta$$

To find α, we use $\Sigma\tau_{\text{ext}} = I\alpha$, where I was obtained in part (A):

$$\alpha = \frac{\sum \tau_{\text{ext}}}{I} = \boxed{\frac{2(m_f - m_d)g\cos\theta}{\ell\left(\frac{M}{3} + m_f + m_d\right)}}$$

Generally, fathers are more massive than daughters, so the angular acceleration is positive. If the seesaw begins in a horizontal orientation ($\theta = 0$) and is released, the rotation will be counterclockwise in Figure 11.9 and the father's end of the seesaw drops. This is consistent with everyday experience.

What If? **After several complaints from the daughter that she simply rises into the air rather than moving up and down as planned, the father moves inward on the seesaw to try to balance the two sides. He moves in to a position that is a distance *d* from the pivot. What is the angular acceleration of the system in this case when it is released from an arbitrary angle θ?**

Answer The angular acceleration of the system should decrease if the system is more balanced. As the father continues to slide inward, he should reach a point at which the seesaw is balanced and there is no angular acceleration of the system when released.

The total moment of inertia about the z axis through O for the modified system is

$$I = \tfrac{1}{12}M\ell^2 + m_f d^2 + m_d\left(\frac{\ell}{2}\right)^2$$

$$= \frac{\ell^2}{4}\left(\frac{M}{3} + m_d\right) + m_f d^2$$

The net torque exerted on the system about O is

$$\tau_{\text{net}} = \tau_f + \tau_d = m_f g d\cos\theta - \tfrac{1}{2}m_d g\ell\cos\theta$$

Now, the angular acceleration of the system is

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{m_f g d\cos\theta - \frac{1}{2}m_d g\ell\cos\theta}{\frac{\ell^2}{4}[(M/3) + m_d] + m_f d^2}$$

The seesaw will be balanced when the angular acceleration is zero. In this situation, both father and daughter can push off the ground and rise to the highest possible point. We find the required position of the father by setting $\alpha = 0$:

$$\alpha = \frac{m_f g d\cos\theta - \frac{1}{2}m_d g\ell\cos\theta}{(\ell^2/4)[(M/3) + m_d] + m_f d^2} = 0$$

$$m_f g d\cos\theta - \tfrac{1}{2}m_d g\ell\cos\theta = 0$$

$$d = \left(\frac{m_d}{m_f}\right)\tfrac{1}{2}\ell$$

In the rare case that the father and daughter have the same mass, the father is located at the end of the seesaw, $d = \ell/2$.

Example 11.7 Formation of a Neutron Star

A star rotates with a period of 30 days about an axis through its center. After the star undergoes a supernova explosion, the stellar core, which had a radius of 1.0×10^4 km, collapses into a neutron star of radius 3.0 km. Determine the period of rotation of the neutron star.

Solution The same physics that makes a skater spin faster with his arms pulled in describes the motion of the neutron star. Let us assume that during the collapse of the stellar core, (1) no external torque acts on it, (2) it remains spherical with the same relative mass distribution, and (3) its mass remains constant. Also, let us use the symbol T for the period, with T_i being the initial period of the star and T_f being the period of the neutron star. The period is the length of time a point on the star's equator takes to make one complete circle around the axis of rotation. The angular speed of the star is given by $\omega = 2\pi/T$. Therefore, Equation 11.19 gives

$$I_i\omega_i = I_f\omega_f$$

$$I_i\left(\frac{2\pi}{T_i}\right) = I_f\left(\frac{2\pi}{T_f}\right)$$

We don't know the mass distribution of the star, but we have assumed that the distribution is symmetric, so that the moment of inertia can be expressed as kMR^2, where k is some numerical constant. (From Table 10.2, for example, we see that $k = 2/5$ for a solid sphere and $k = 2/3$ for a spherical shell.) Thus, we can rewrite the preceding equation as

$$kMR_i^2\left(\frac{2\pi}{T_i}\right) = kMR_f^2\left(\frac{2\pi}{T_f}\right)$$

$$T_f = \left(\frac{R_f^2}{R_i^2}\right)T_i$$

Substituting numerical values gives

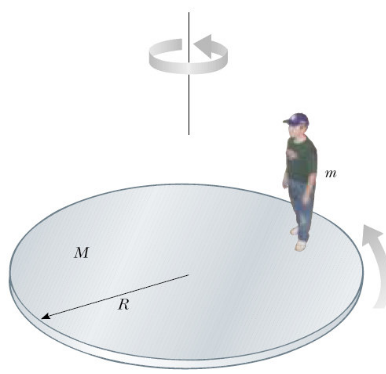
$$T_f = (30\text{ days})\left(\frac{3.0\text{ km}}{1.0\times10^4\text{ km}}\right)^2 = 2.7\times10^{-6}\text{ days}$$

$$= \boxed{0.23\text{ s}}$$

Thus, the neutron star rotates about four times each second.

Example 11.8 The Merry-Go-Round

A horizontal platform in the shape of a circular disk rotates freely in a horizontal plane about a frictionless vertical axle (Fig. 11.11). The platform has a mass $M = 100$ kg and a radius $R = 2.0$ m. A student whose mass is $m = 60$ kg walks slowly from the rim of the disk toward its center. If the angular speed of the system is 2.0 rad/s when the student is at the rim, what is the angular speed when he reaches a point $r = 0.50$ m from the center?



Solution The speed change here is similar to the increase in angular speed of the spinning skater when he pulls his arms inward. Let us denote the moment of inertia of the platform as I_p and that of the student as I_s. Modeling the student as a particle, we can write the initial moment of inertia I_i of the system (student plus platform) about the axis of rotation:

$$I_i = I_{pi} + I_{si} = \tfrac{1}{2}MR^2 + mR^2$$

When the student walks to the position $r < R$, the moment of inertia of the system reduces to

$$I_f = I_{pf} + I_{sf} = \tfrac{1}{2}MR^2 + mr^2$$

Note that we still use the greater radius R when calculating I_{pf} because the radius of the platform does not change. Because no external torques act on the system about the axis of rotation, we can apply the law of conservation of angular momentum:

$$I_i\omega_i = I_f\omega_f$$

$$(\tfrac{1}{2}MR^2 + mR^2)\omega_i = (\tfrac{1}{2}MR^2 + mr^2)\omega_f$$

$$\omega_f = \left(\frac{\frac{1}{2}MR^2 + mR^2}{\frac{1}{2}MR^2 + mr^2}\right)\omega_i$$

$$= \left(\frac{\frac{1}{2}(100\text{ kg})(2.0\text{ m})^2 + (60\text{ kg})(2.0\text{ m})^2}{\frac{1}{2}(100\text{ kg})(2.0\text{ m})^2 + (60\text{ kg})(0.50\text{ m})^2}\right)(2.0\text{ rad/s})$$

$$= \left(\frac{440\text{ kg}\cdot\text{m}^2}{215\text{ kg}\cdot\text{m}^2}\right)(2.0\text{ rad/s}) = \boxed{4.1\text{ rad/s}}$$

As expected, the angular speed increases.

What If? **What if we were to measure the kinetic energy of the system before and after the student walks inward? Are they the same?**

Answer You may be tempted to say yes because the system is isolated. But remember that energy comes in several forms, so we have to handle an energy question carefully. The initial kinetic energy is

$$K_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}(440\ \text{kg}\cdot\text{m}^2)(2.0\ \text{rad/s})^2 = 880\ \text{J}$$

The final kinetic energy is

$$K_f = \frac{1}{2}I_f\,\omega_f^2 = \frac{1}{2}(215\ \text{kg}\cdot\text{m}^2)(4.1\ \text{rad/s})^2 = 1.81 \times 10^3\ \text{J}$$

Thus, the kinetic energy of the system *increases*. The student must do work to move himself closer to the center of rotation, so this extra kinetic energy comes from chemical potential energy in the body of the student.