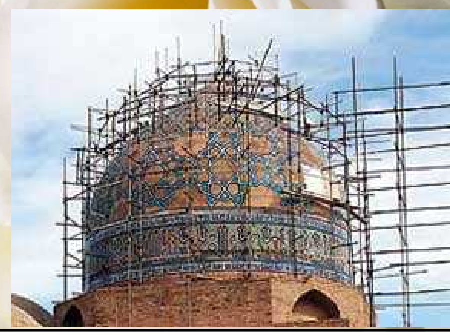
دستگاه های مختصات کارتزی و قطبی

برای تعمیم حرکت یک بعدی ذره به ابعاد بالاتر (دو یا سه بعد) از مفاهیم بردارها و فضاهی برداری استفاده می کنیم

قبل از هر چیز به تقارن دستگاه توجه می کنیم

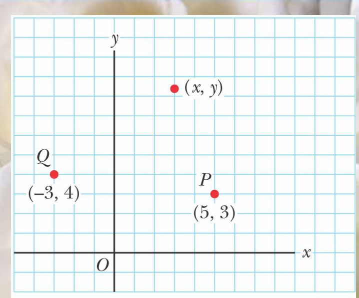
سپس بر مبنای تقارن دستگاه یک دستگاه مختصات مناسب اختیار می کنیم



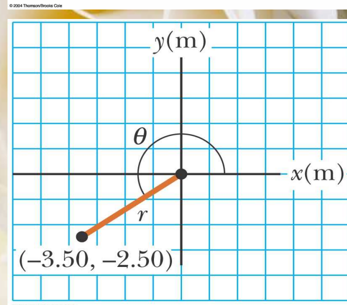
دستگاه های مختصات کارتزی و قطبی

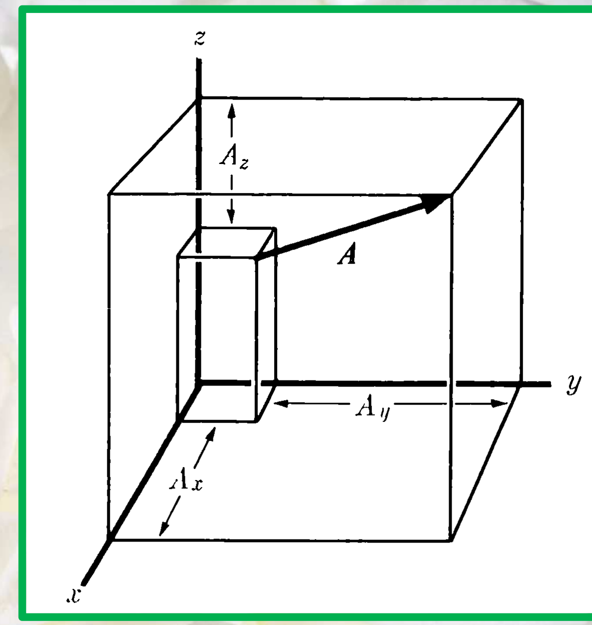
تعیین یک نقطه در دستگاه مختصات کارتزی با دو پارامتر x و y انجام می شود

$P(5,3)$ $\quad$ $Q(-3,4)$



تعیین یک نقط در دستگاه مختصات قطبی با دو پارامتر r و θ انجام می شود





ارتباط دستگاه های مختصات کارتزی و قطبی و نحوه تبدیل آنها به یکدیگر

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$x^2 = r^2\cos^2\theta$$

$$y^2 = r^2\sin^2\theta$$

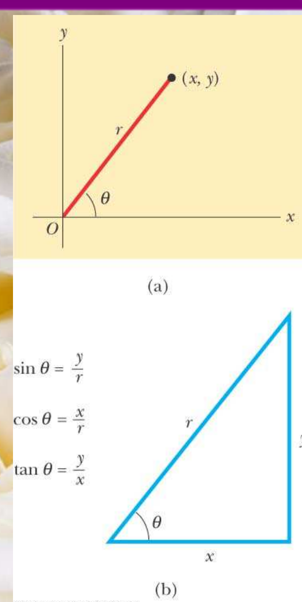
$$x^2 + y^2 = r^2\left(\cos^2\theta + \sin^2\theta\right)$$

$$x^2 + y^2 = r^2$$

$$\sqrt{x^2 + y^2} = r$$

$$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta}$$

$$\frac{y}{x} = \tan\theta$$



بردارها

یک بردار هم داری اندازه است و هم جهت درحالی که یک اسکالر فقط دارای اندازه است

یک بردار را با یک پیکان به صورت مقابل نمایش می دهیم

این بردار می تواند معرف یک کمیت فیزیکی برداری باشد

در این صورت طول این بردار معرف اندازه آن کمیت فیزیکی است

و جهت این بردار معرف جهت آن کمیت فیزیکی است

یک بردار را به صورت زیر نمایش می دهند

$\vec{A}$

دو روش برای نمایش یک بردار وجود دارد

روش اول از طریق اندازه و جهت بردار، یعنی

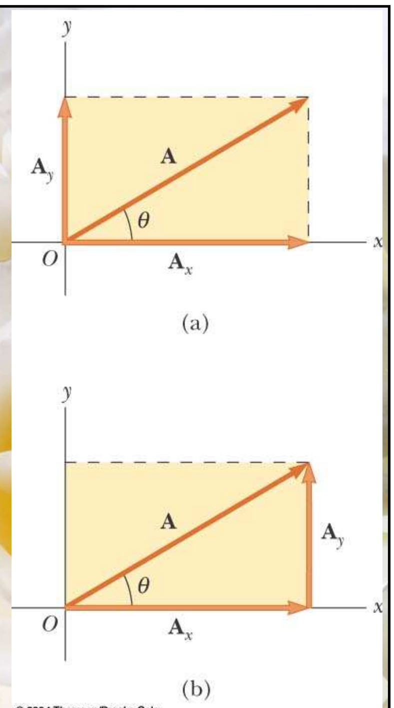
A θ

به این روش، روش هندسی گویند

روش دوم از طریق مولفه های آن، یعنی

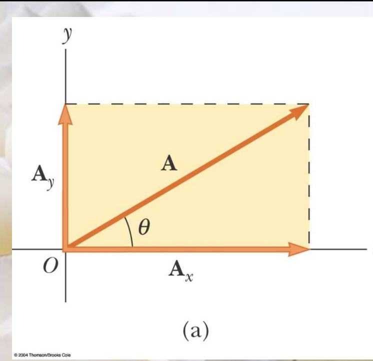
A_x A_y

به این روش، روش جبری گویند



روش هندسی نمایش بردار هم ارز روش جبری است

این دو روش قابل تبدیل به یکدیگر هستند:



$$\vec{A} = A\cos\theta\hat{\imath} + A\sin\theta\hat{\jmath}$$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$A_x = A\cos\theta$$

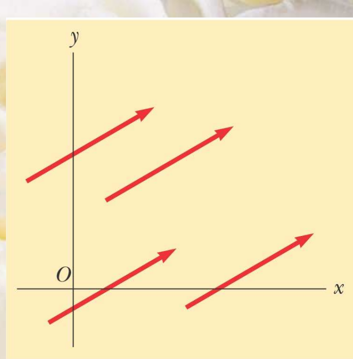
$$A_y = A\sin\theta$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A_y}{A_x}$$

توجه شود که موقعیت بردارها می تواند نامعین باشد

یعنی بردارهای زیر هم ارز یکد یگر هستند



یک کمیت اسکالری فقط دارای اندازه است و جهت ندارد

در زیر چند کمیت اسکالری به عنوان مثال ارایه شده است

1) دما

2) جرم

3) سن یک شخص

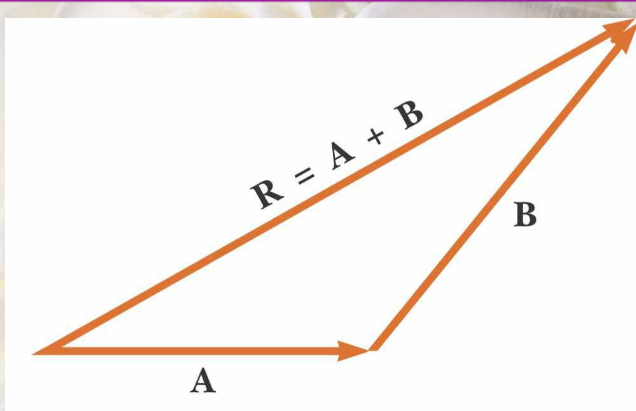
4) جریان الکتریکی

جمع برداری به دو صورت قابل انجام است

1) روش هندسی

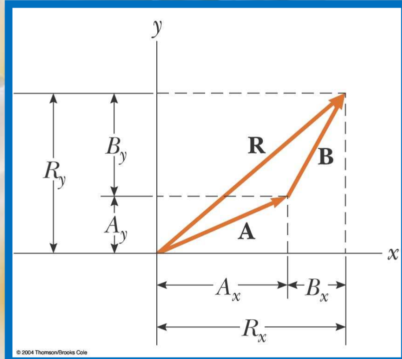
2) روش جبری

روش هندسی در شکل زیر نمایش داده شده است

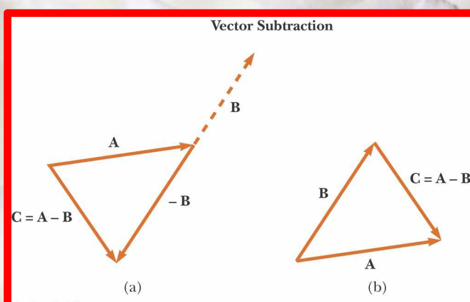


روش جبری جمع بردارها

$$\begin{aligned}\mathbf{R} &= \mathbf{A}+\mathbf{B}\\ &= \left(A_x\hat{\mathbf{i}}+A_y\hat{\mathbf{j}}\right)+\left(B_x\hat{\mathbf{i}}+B_y\hat{\mathbf{j}}\right)\\ &= \left(A_x+B_x\right)\hat{\mathbf{i}}+\left(A_y+B_y\right)\hat{\mathbf{j}}\\ &= R_x\hat{\mathbf{i}}+R_y\hat{\mathbf{j}}\end{aligned}$$



روش هندسی تفاضل بردارها

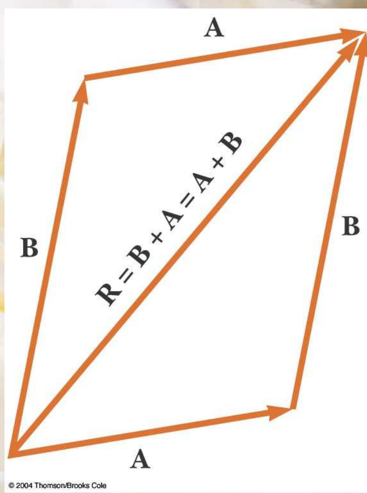


روش جبری تفاضل بردارها

$$\boldsymbol{A}-\boldsymbol{B} = \boldsymbol{A}+(-\boldsymbol{B}) = (A_x-B_x,\ A_y-B_y,\ A_z-B_z)$$

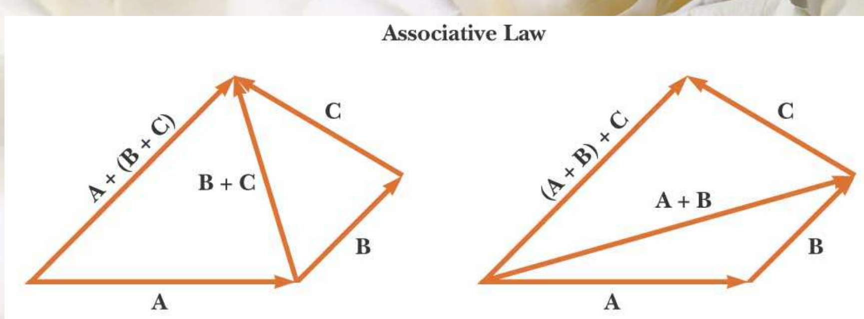
جمع برداری دارای خاصیت جابه جایی است

اثبات هندسی خاصیت جابه جایی در شکل زیر انجام شده است

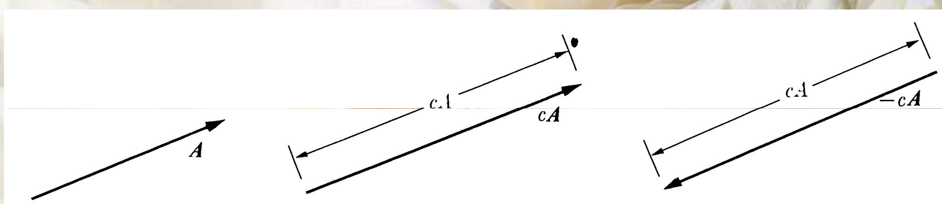


جمع برداری دارای خاصیت شرکت پذیری است

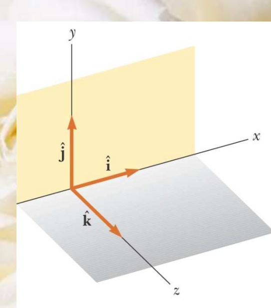
اثبات هندسی خاصیت شرکت پذیری در شکلهای زیر انجام شده است



ضرب اسکالری



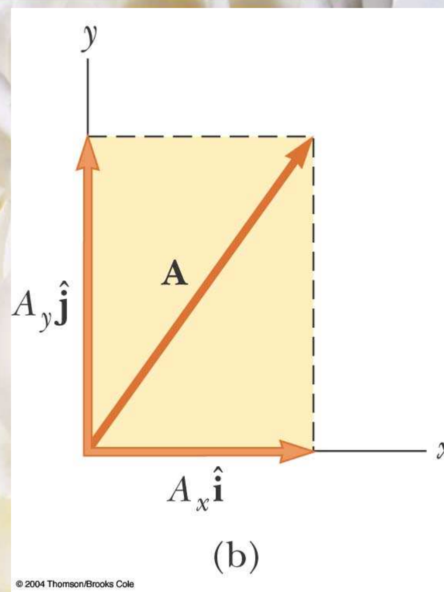
بردارهای یکه همان طور که از نامشان برمی آید دارای طول واحد هستند



(a)

با استفاده از بردارهای یکه می توان یک بردار را به مولفه های آن تجزیه کرد

$$\boxed{\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}}$$



بردارها را می توان به دو روش در هم ضرب کرد

1) ضرب داخلی

2) ضرب خارجی

ضرب داخلی را می توان به دو روش انجام داد

الف) روش هندسی

ب) روش جبری

ضرب خارجی را می توان به دو روش انجام داد

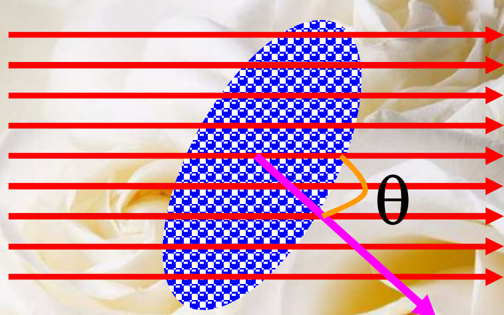
الف) روش هندسی

ب) روش جبری

روش جبری ضرب داخلی

$$C=\vec{A}.\vec{B}$$
$$=\left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right).\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$$
$$=A_xB_x+A_yB_y+A_zB_z$$

روش هندسی ضرب داخلی



$$C=\vec{A}.\vec{B}$$
$$=\left|\vec{A}\right|.\left|\vec{B}\right|\cos(\vec{A},\vec{B})=AB\cos\theta$$

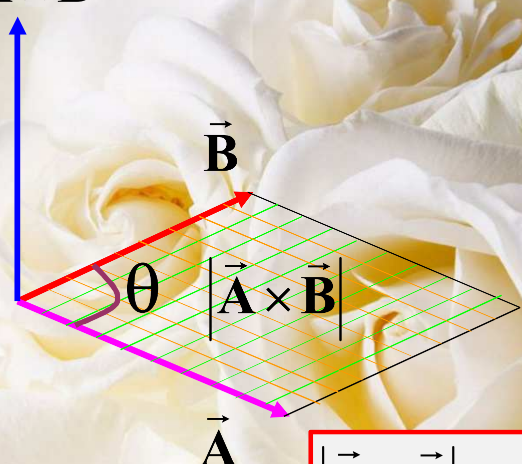
روش جبری ضرب خارجی

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} =$$

$$= \hat{i}\left(A_y B_z - A_z B_y\right) - \hat{j}\left(A_x B_z - A_z B_x\right) + \hat{k}\left(A_x B_y - A_y B_x\right)$$

روش هندسی ضرب خارجی

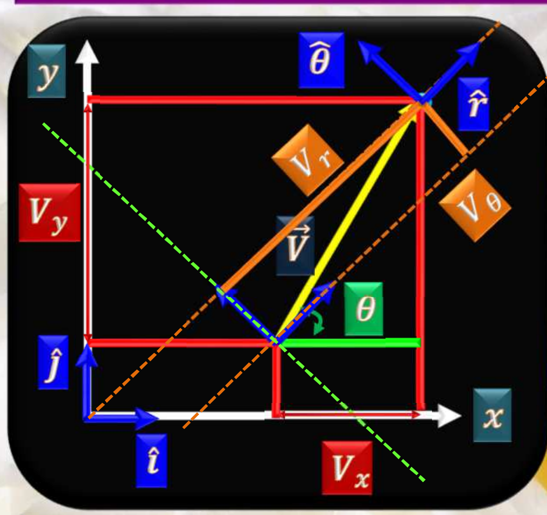
$$\vec{C} = \vec{A} \times \vec{B}$$



$$\left|\vec{A} \times \vec{B}\right| = AB \sin\theta$$

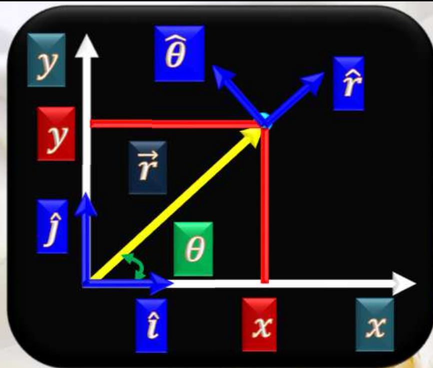
$$A\cdot(B\times C) = (A\times B)\cdot C,$$
$$A\times(B\times C) = B(A\cdot C) - C(A\cdot B),$$
$$(A\times B)\times C = B(A\cdot C) - A(B\cdot C),$$

1 و 2) یک بردار در دستگاه های مختصات کارتزی 2 بعدی و قطبی



$$\vec{V} = V_x\hat{\imath} + V_y\hat{\jmath}$$
$$\vec{V} = V_r\hat{r} + V_\theta\hat{\theta}$$
$$V_x = V\cos\theta$$
$$V_y = V\sin\theta$$
$$V_x^2 + V_y^2 = V^2$$
$$\frac{V_y}{V_x} = \tan\theta$$

بردار مکان در دستگاه های کارتزی و قطبی



$$\vec{V} = V_r\hat{r} + V_\theta\hat{\theta}$$

$$if\ \vec{V} = \vec{r} \Rightarrow V_\theta = 0,\quad V_r = r$$

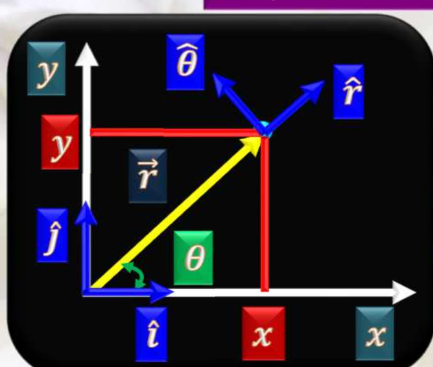
$$\vec{r} = r\hat{r}$$

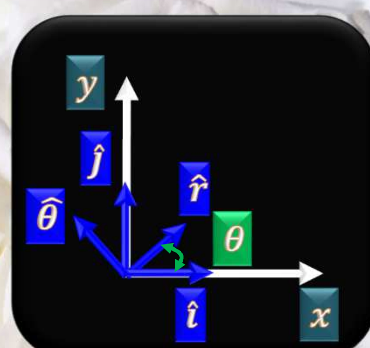
پس بردار مکان شکل ساده ای در دستگاه قطبی دارد

به همین دلیل مسایلی که دارای تقارن دایروی هستند در دستگاه قطبی راحتر حل می شوند

تذکر: بدیهی است که بین بردار یکه $\hat{\theta}$ و زاویهθتفاوت اساسی وجود دارد!

ارتباط بردارهای یکه قطبی با بردارهای یکه کارتزی





از شکل به راحتی پیداست که:

$$\hat{r} = cos\theta\hat{\imath} + \sin\theta\hat{\jmath}$$

$$\hat{\theta} = -sin\theta\hat{\imath} + \cos\theta\hat{\jmath}$$

نکته: بردارهای یکه قطبی با تغییر زاویه (حرکت دورانی ذره) تغییر می کنند ولی بردارهای یکه کارتزی ثابتند

پس بردارهای یکه قطبی یک ذره در یک مسیر منحنی الشکل تابع زمانند ولی بردارهای یکه کارتزی مستقل از زمان

ارتباط بردارهای یکه قطبی با یکدیگر:

با مشتق گیری از بردار یکه $\hat{r}$ نسبت به زاویه θ

$$\hat{r} = cos\theta\hat{\imath} + \sin\theta\hat{\jmath}$$

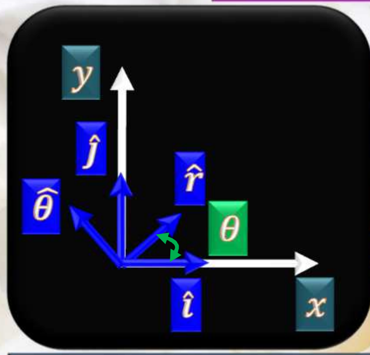
خواهیم داشت که:

$$\frac{d\hat{r}}{d\theta} = -sin\theta\hat{\imath} + \cos\theta\hat{\jmath}$$

که چیزی جز بردار یکه θ نیست

$$\frac{d\hat{r}}{d\theta} = \hat{\theta}$$

به همین ترتیب می توان نشان داد که:



$$\frac{d\hat{\theta}}{d\theta} = -cos\theta\hat{\imath} - \sin\theta\hat{\jmath}$$

که چیزی جز منفی بردار یکه $\hat{r}$ نیست

$$\frac{d\hat{\theta}}{d\theta} = -\hat{r}$$

تغییرات زمانی بردارهای یکه قطبی

$$\frac{d\hat{\theta}}{dt} = \frac{d\hat{\theta}}{d\theta}\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \dot{\theta} = \omega$$

$$\frac{d\hat{r}}{dt} = \frac{d\hat{r}}{d\theta}\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \dot{\theta} = \omega$$

که چیزی جز سرعت زاویه ای نیست

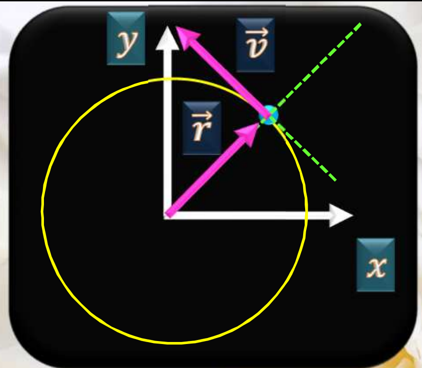
$$\frac{d\hat{\theta}}{d\theta} = -\hat{r}$$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r} = -\omega\hat{r}$$

$$\frac{d\hat{r}}{d\theta} = \hat{\theta}$$

$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta} = \omega\hat{\theta}$$

بردار سرعت در حرکت دایره ای یکنواخت



از شکل پیداست که بردار مکان فقط مولفه شعاعی و بردار سرعت فقط مولفه مماسی دارد:

$\vec{r} = R\hat{r}$ $\quad$ $\vec{v} = v_\theta\hat{\theta}$

به صورت تحلیلی نیز می توان نشان داد که در این حالت سرعت فقط دارای مولفه مماسی است

$$\vec{v} = \frac{d\vec{r}}{dt} = v_r\hat{r} + v_\theta\hat{\theta}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(R\hat{r})$$

$$\vec{v} = \dot{R}\hat{r} + R\frac{d\hat{r}}{dt}$$

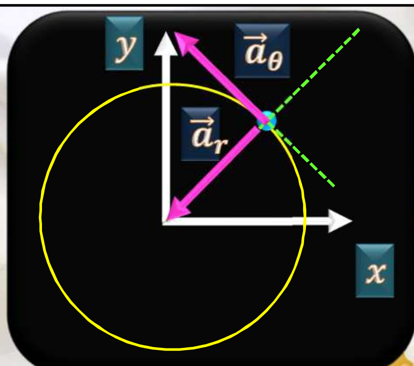
$$\vec{v} = R\dot{\theta}\hat{\theta} = R\omega\hat{\theta}$$

$$\vec{v} = v_r\hat{r} + v_\theta\hat{\theta} = R\omega\hat{\theta}$$

$v_r = 0$ $\quad$ $v_\theta = R\omega$ $= v$

در حرکت دایره ای یکنواخت معمولا اندیس سرعت را نمی گذارند (زیرا فقط یک اندیس مماسی دارد)

بردار شتاب در حرکت دایره ای یکنواخت



$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(R\dot{\theta}\hat{\theta})}{dt}$$

$$\vec{a} = R\ddot{\theta}\hat{\theta} + R\dot{\theta}\frac{d\hat{\theta}}{dt}$$

$$\vec{a} = R\ddot{\theta}\hat{\theta} + R\dot{\theta}(-\dot{\theta}\hat{r})$$

$$\vec{a} = R\ddot{\theta}\hat{\theta} - R\dot{\theta}^2\hat{r}$$

$$\vec{a} = a_r\hat{r} + a_\theta\hat{\theta} = -R\dot{\theta}^2\hat{r} + R\ddot{\theta}\hat{\theta}$$

$a_r = -R\dot{\theta}^2$ $\quad$ $a_\theta = R\ddot{\theta}$

$a_r = -R\omega^2$ $= -R\frac{v^2}{R^2} = -\frac{v^2}{R}$

$v = R\omega$

1) بردار مکان همواره (چه در حرکت دایره ای یکنواخت و چه در حرکت دایره ای غیر یکنواخت) دارای مولفه شعاعی است

$$\vec{r} = R\hat{r}$$

2) بردار سرعت در حرکت دایره ای یکنواخت فقط دارای مولفه مماسی است

$$\vec{v} = v_\theta\hat{\theta} = R\dot{\theta}\hat{\theta} = R\omega\hat{\theta}$$

3) بردار شتاب در حرکت دایره ای یکنواخت دارای هر دو مولفه شعاعی و مماسی است

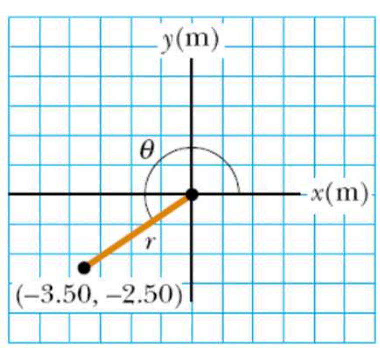
$$\vec{a} = a_r\hat{r} + a_\theta\hat{\theta} = -R\dot{\theta}^2\hat{r} + R\ddot{\theta}\hat{\theta} = -R\omega^2\hat{r} + R\ddot{\theta}\hat{\theta} = -\frac{v^2}{R}\hat{r} + R\ddot{\theta}\hat{\theta}$$

مولفه شعاعی شتاب همان شتاب جانب مرکز است

Example 3.1 Polar Coordinates

The Cartesian coordinates of a point in the xy plane are $(x, y) = (-3.50, -2.50)$ m, as shown in Figure 3.3. Find the polar coordinates of this point.

Solution For the examples in this and the next two chapters we will illustrate the use of the General Problem-Solving



Strategy outlined at the end of Chapter 2. In subsequent chapters, we will make fewer explicit references to this strategy, as you will have become familiar with it and should be applying it on your own. The drawing in Figure 3.3 helps us to *conceptualize* the problem. We can *categorize* this as a plug-in problem. From Equation 3.4,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50 \text{ m})^2 + (-2.50 \text{ m})^2} = \boxed{4.30 \text{ m}}$$

and from Equation 3.3,

$$\tan \theta = \frac{y}{x} = \frac{-2.50 \text{ m}}{-3.50 \text{ m}} = 0.714$$

$$\theta = \boxed{216°}$$

Note that you must use the signs of x and y to find that the point lies in the third quadrant of the coordinate system. That is, $\theta = 216°$ and not $35.5°$.

Quick Quiz 3.1 Which of the following are vector quantities and which are scalar quantities? (a) your age (b) acceleration (c) velocity (d) speed (e) mass

Quick Quiz 3.2 The magnitudes of two vectors **A** and **B** are $A = 12$ units and $B = 8$ units. Which of the following pairs of numbers represents the *largest* and *smallest* possible values for the magnitude of the resultant vector **R** = **A** + **B**? (a) 14.4 units, 4 units (b) 12 units, 8 units (c) 20 units, 4 units (d) none of these answers.

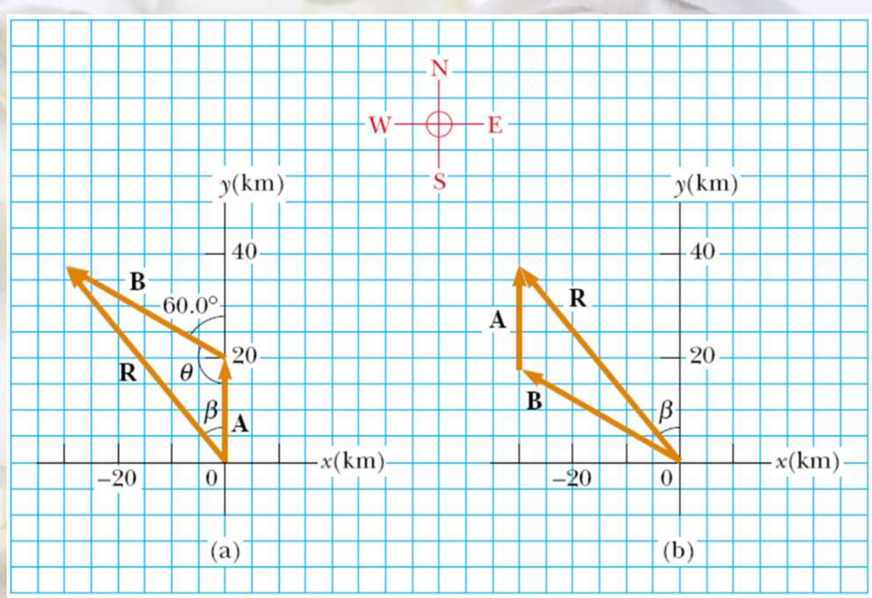
Quick Quiz 3.3 If vector **B** is added to vector **A**, under what condition does the resultant vector **A** + **B** have magnitude $A + B$? (a) **A** and **B** are parallel and in the same direction. (b) **A** and **B** are parallel and in opposite directions. (c) **A** and **B** are perpendicular.

Quick Quiz 3.4 If vector **B** is added to vector **A**, which *two* of the following choices must be true in order for the resultant vector to be equal to zero? (a) **A** and **B** are parallel and in the same direction. (b) **A** and **B** are parallel and in opposite directions. (c) **A** and **B** have the same magnitude. (d) **A** and **B** are perpendicular.

Example 3.2 A Vacation Trip

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north, as shown in Figure 3.12a. Find the magnitude and direction of the car's resultant displacement.

Solution The vectors **A** and **B** drawn in Figure 3.12a help us to *conceptualize* the problem. We can *categorize* this as a relatively simple analysis problem in vector addition. The displacement **R** is the resultant when the two individual displacements **A** and **B** are added. We can further categorize this as a problem about the analysis of triangles, so we appeal to our expertise in geometry and trigonometry.



In this example, we show two ways to *analyze* the problem of finding the resultant of two vectors. The first way is to solve the problem geometrically, using graph paper and a protractor to measure the magnitude of **R** and its direction in Figure 3.12a. (In fact, even when you know you are going to be carrying out a calculation, you should sketch the vectors to check your results.) With an ordinary ruler and protractor, a large diagram typically gives answers to two-digit but not to three-digit precision.

The second way to solve the problem is to analyze it algebraically. The magnitude of **R** can be obtained from the law of cosines as applied to the triangle (see Appendix B.4). With $\theta = 180^\circ - 60^\circ = 120^\circ$ and $R^2 = A^2 + B^2 - 2AB\cos\theta$, we find that

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$= \sqrt{(20.0\text{ km})^2 + (35.0\text{ km})^2 - 2(20.0\text{ km})(35.0\text{ km})\cos 120^\circ}$$

$$= \boxed{48.2\text{ km}}$$

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta = \frac{35.0\text{ km}}{48.2\text{ km}}\sin 120° = 0.629$$

$$\beta = 39.0°$$

The resultant displacement of the car is 48.2 km in a direction 39.0° west of north.

What If? Suppose the trip were taken with the two vectors in reverse order: 35.0 km at 60.0° west of north first, and then 20.0 km due north. How would the magnitude and the direction of the resultant vector change?

Answer They would not change. The commutative law for vector addition tells us that the order of vectors in an addition is irrelevant. Graphically, Figure 3.12b shows that the vectors added in the reverse order give us the same resultant vector.

Quick Quiz 3.5 Choose the correct response to make the sentence true: A component of a vector is (a) always, (b) never, or (c) sometimes larger than the magnitude of the vector.

Quick Quiz 3.6 If at least one component of a vector is a positive number, the vector cannot (a) have any component that is negative (b) be zero (c) have three dimensions.

Quick Quiz 3.7 If $\mathbf{A} + \mathbf{B} = 0$, the corresponding components of the two vectors $\mathbf{A}$ and $\mathbf{B}$ must be (a) equal (b) positive (c) negative (d) of opposite sign.

Quick Quiz 3.8 For which of the following vectors is the magnitude of the vector equal to one of the components of the vector? (a) $\mathbf{A} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$ (b) $\mathbf{B} = -3\hat{\mathbf{j}}$ (c) $\mathbf{C} = +5\hat{\mathbf{k}}$

Example 3.3 The Sum of Two Vectors

Find the sum of two vectors $\mathbf{A}$ and $\mathbf{B}$ lying in the *xy* plane and given by

$$\mathbf{A} = (2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\text{ m} \qquad \text{and} \qquad \mathbf{B} = (2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}})\text{ m}$$

Solution You may wish to draw the vectors to *conceptualize* the situation. We *categorize* this as a simple plug-in problem. Comparing this expression for $\mathbf{A}$ with the general expression $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$, we see that $A_x = 2.0$ m and $A_y = 2.0$ m. Likewise, $B_x = 2.0$ m and $B_y = -4.0$ m. We obtain the resultant vector $\mathbf{R}$, using Equation 3.14:

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (2.0 + 2.0)\hat{\mathbf{i}}\text{ m} + (2.0 - 4.0)\hat{\mathbf{j}}\text{ m}$$
$$= (4.0\hat{\mathbf{i}} - 2.0\hat{\mathbf{j}})\text{ m}$$

or

$$R_x = 4.0\text{ m} \qquad R_y = -2.0\text{ m}$$

The magnitude of $\mathbf{R}$ is found using Equation 3.16:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0 \text{ m})^2 + (-2.0 \text{ m})^2} = \sqrt{20} \text{ m}$$

$$= \boxed{4.5 \text{ m}}$$

We can find the direction of $\mathbf{R}$ from Equation 3.17:

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2.0 \text{ m}}{4.0 \text{ m}} = -0.50$$

Your calculator likely gives the answer $-27°$ for $\theta = \tan^{-1}(-0.50)$. This answer is correct if we interpret it to mean 27° clockwise from the x axis. Our standard form has been to quote the angles measured counterclockwise from the $+x$ axis, and that angle for this vector is $\boxed{\theta = 333°}$.

Example 3.4 The Resultant Displacement

A particle undergoes three consecutive displacements: $\mathbf{d}_1 = (15\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$ cm, $\mathbf{d}_2 = (23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}})$ cm and $\mathbf{d}_3 = (-13\hat{\mathbf{i}} + 15\hat{\mathbf{j}})$ cm. Find the components of the resultant displacement and its magnitude.

Solution Three-dimensional displacements are more difficult to imagine than those in two dimensions, because the latter can be drawn on paper. For this problem, let us *conceptualize* that you start with your pencil at the origin of a piece of graph paper on which you have drawn x and y axes. Move your pencil 15 cm to the right along the x axis, then 30 cm upward along the y axis, and then 12 cm *vertically away* from the graph paper. This provides the displacement described by $\mathbf{d}_1$. From this point, move your pencil 23 cm to the right parallel to the x axis, 14 cm parallel to the graph paper in the $-y$ direction, and then 5.0 cm vertically downward toward the graph paper. You are now at the displacement from the origin described by $\mathbf{d}_1 + \mathbf{d}_2$. From this point, move your pencil 13 cm to the left in the $-x$ direction, and (finally!) 15 cm parallel to the graph paper along the y axis.

Your final position is at a displacement $\mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3$ from the origin.

Despite the difficulty in conceptualizing in three dimensions, we can *categorize* this problem as a plug-in problem due to the careful bookkeeping methods that we have developed for vectors. The mathematical manipulation keeps track of this motion along the three perpendicular axes in an organized, compact way:

$$\begin{aligned}\mathbf{R} &= \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3\\ &= (15 + 23 - 13)\hat{\mathbf{i}}\text{ cm} + (30 - 14 + 15)\hat{\mathbf{j}}\text{ cm}\\ &\quad + (12 - 5.0 + 0)\hat{\mathbf{k}}\text{ cm}\\ &= (25\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 7.0\hat{\mathbf{k}})\text{ cm}\end{aligned}$$

The resultant displacement has components $R_x = 25$ cm, $R_y = 31$ cm, and $R_z = 7.0$ cm. Its magnitude is

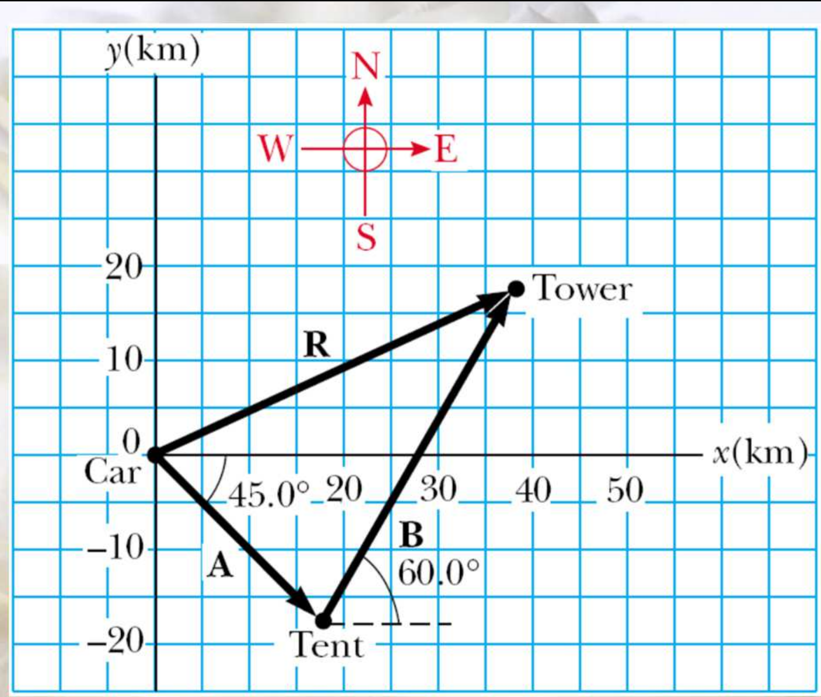
$$\begin{aligned}R &= \sqrt{R_x^2 + R_y^2 + R_z^2}\\ &= \sqrt{(25\text{ cm})^2 + (31\text{ cm})^2 + (7.0\text{ cm})^2} = \boxed{40\text{ cm}}\end{aligned}$$

Example 3.5 Taking a Hike

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.

Solution We *conceptualize* the problem by drawing a sketch as in Figure 3.19. If we denote the displacement vectors on the first and second days by **A** and **B**, respectively, and use the car as the origin of coordinates, we obtain the vectors shown in Figure 3.19. Drawing the resultant **R**, we can now *categorize* this as a problem we've solved before—an addition of two vectors. This should give you a hint of the power of categorization—many new problems are very similar to problems that we have already solved if we are careful to conceptualize them.



We will *analyze* this problem by using our new knowledge of vector components. Displacement **A** has a magnitude of 25.0 km and is directed 45.0° below the positive x axis. From Equations 3.8 and 3.9, its components are

$$A_x = A\cos(-45.0°) = (25.0\text{ km})(0.707) = \boxed{17.7\text{ km}}$$

$$A_y = A\sin(-45.0°) = (25.0\text{ km})(-0.707) = \boxed{-17.7\text{ km}}$$

The negative value of A_y indicates that the hiker walks in the negative y direction on the first day. The signs of A_x and A_y also are evident from Figure 3.19.

The second displacement **B** has a magnitude of 40.0 km and is 60.0° north of east. Its components are

$$B_x = B\cos 60.0^\circ = (40.0\text{ km})(0.500) = \boxed{20.0\text{ km}}$$

$$B_y = B\sin 60.0^\circ = (40.0\text{ km})(0.866) = \boxed{34.6\text{ km}}$$

(B) Determine the components of the hiker's resultant displacement **R** for the trip. Find an expression for **R** in terms of unit vectors.

Solution The resultant displacement for the trip **R** = **A** + **B** has components given by Equation 3.15:

$$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km} = \boxed{37.7\text{ km}}$$

$$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km} = \boxed{16.9\text{ km}}$$

In unit–vector form, we can write the total displacement as

$$\mathbf{R} = \boxed{(37.7\hat{\mathbf{i}} + 16.9\hat{\mathbf{j}})\text{ km}}$$

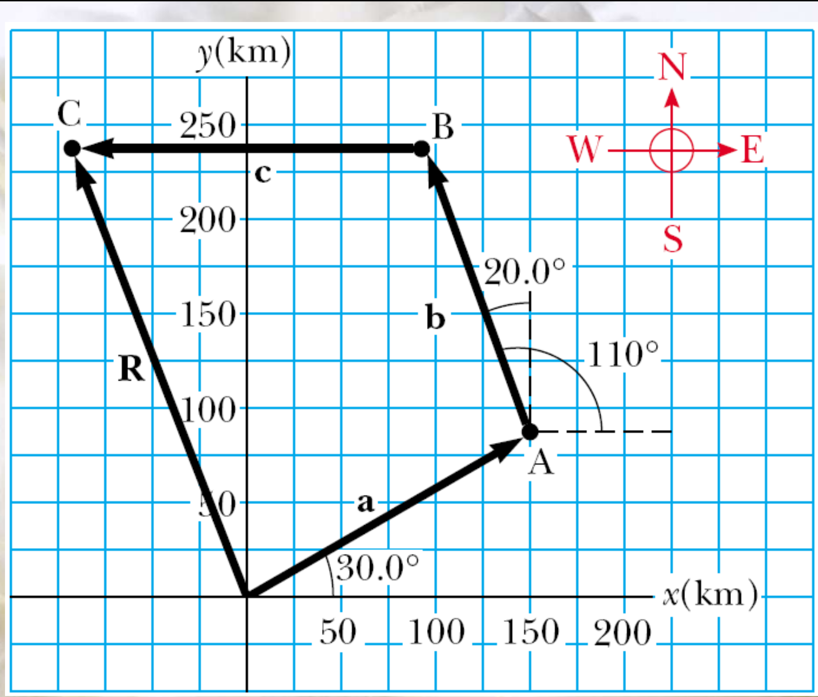
Using Equations 3.16 and 3.17, we find that the vector **R** has a magnitude of 41.3 km and is directed 24.1° north of east.

Let us *finalize.* The units of **R** are km, which is reasonable for a displacement. Looking at the graphical representation in Figure 3.19, we estimate that the final position of the hiker is at about (38 km, 17 km) which is consistent with the components of **R** in our final result. Also, both components of **R** are positive, putting the final position in the first quadrant of the coordinate system, which is also consistent with Figure 3.19.

Example 3.6 Let's Fly Away!

A commuter airplane takes the route shown in Figure 3.20. First, it flies from the origin of the coordinate system shown to city A, located 175 km in a direction 30.0° north of east. Next, it flies 153 km 20.0° west of north to city B. Finally, it flies 195 km due west to city C. Find the location of city C relative to the origin.

Solution Once again, a drawing such as Figure 3.20 allows us to *conceptualize* the problem. It is convenient to choose the coordinate system shown in Figure 3.20, where the x axis points to the east and the y axis points to the north. Let us denote the three consecutive displacements by the vectors **a**, **b**, and **c**.



We can now *categorize* this problem as being similar to Example 3.5 that we have already solved. There are two primary differences. First, we are adding three vectors instead of two. Second, Example 3.5 guided us by first asking for the components in part (A). The current Example has no such guidance and simply asks for a result. We need to *analyze* the situation and choose a path. We will follow the same pattern that we did in Example 3.5, beginning with finding the components of the three vectors **a**, **b**, and **c**. Displacement **a** has a magnitude of 175 km and the components

$$a_x = a\cos(30.0°) = (175\text{ km})(0.866) = 152\text{ km}$$

$$a_y = a\sin(30.0°) = (175\text{ km})(0.500) = 87.5\text{ km}$$

Displacement **b**, whose magnitude is 153 km, has the components

$$b_x = b\cos(110°) = (153\text{ km})(-0.342) = -52.3\text{ km}$$

$$b_y = b\sin(110°) = (153\text{ km})(0.940) = 144\text{ km}$$

Finally, displacement **c,** whose magnitude is 195 km, has the components

$$c_x = c\cos(180°) = (195\text{ km})(-1) = -195\text{ km}$$

$$c_y = c\sin(180°) = 0$$

Therefore, the components of the position vector **R** from the starting point to city C are

$$R_x = a_x + b_x + c_x = 152\text{ km} - 52.3\text{ km} - 195\text{ km}$$

$$= -95.3\text{ km}$$

$$R_y = a_y + b_y + c_y = 87.5\text{ km} + 144\text{ km} + 0 = 232\text{ km}$$

In unit–vector notation, $\mathbf{R} = (-95.3\hat{\mathbf{i}} + 232\hat{\mathbf{j}})\text{ km}$. Using Equations 3.16 and 3.17, we find that the vector **R** has a magnitude of 251 km and is directed 22.3° west of north.

To *finalize* the problem, note that the airplane can reach city C from the starting point by first traveling 95.3 km due west and then by traveling 232 km due north. Or it could follow a straight-line path to C by flying a distance $R = 251$ km in a direction 22.3° west of north.

What If? **After landing in city C, the pilot wishes to return to the origin along a single straight line. What are the components of the vector representing this displacement? What should the heading of the plane be?**

Answer The desired vector $\mathbf{H}$ (for Home!) is simply the negative of vector $\mathbf{R}$:

$$\mathbf{H} = -\mathbf{R} = (+95.3\hat{\mathbf{i}} - 232\hat{\mathbf{j}}) \text{ km}$$

The heading is found by calculating the angle that the vector makes with the x axis:

$$\tan\theta = \frac{R_y}{R_x} = \frac{-232 \text{ m}}{95.3 \text{ m}} = -2.43$$

This gives a heading angle of $\theta = -67.7°$, or 67.7° south of east.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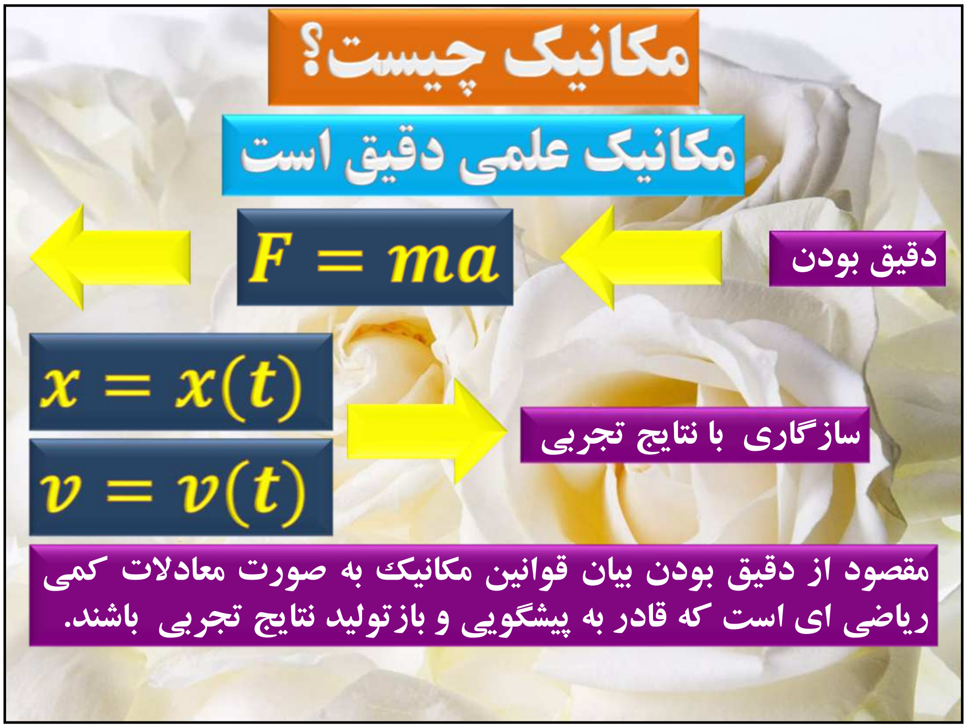
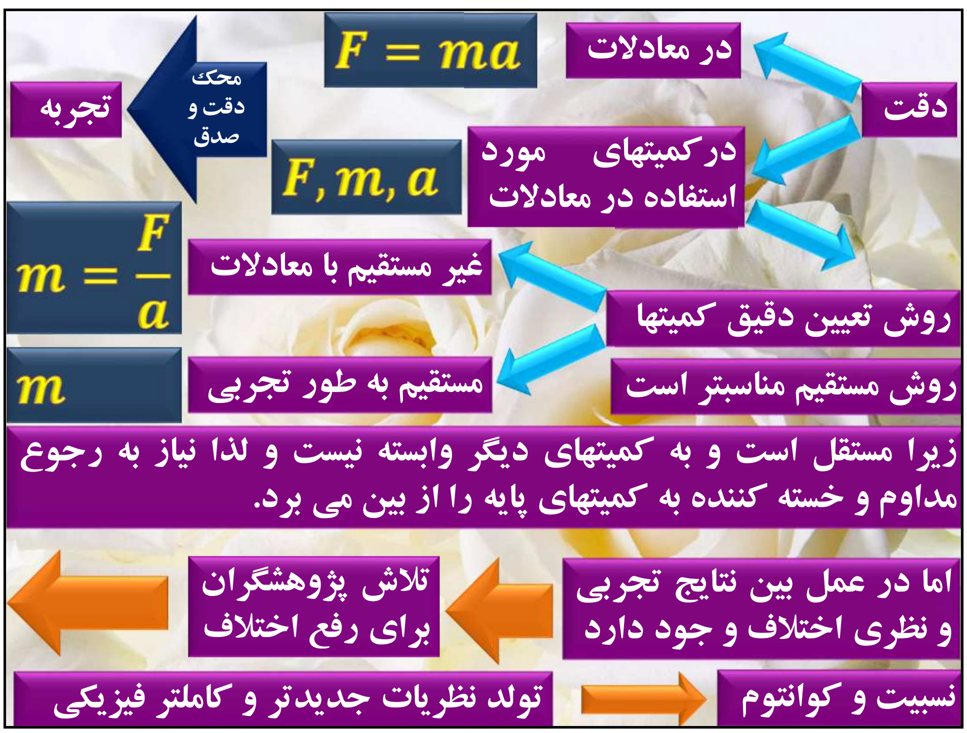
PHYSICS
BIOLOGY


مکانیک چیست؟
مکانیک علمی دقیق است
دقیق بودن
$F = ma$
سازگاری با نتایج تجربی
$x = x(t)$
$v = v(t)$
مقصود از دقیق بودن بیان قوانین مکانیک به صورت معادلات کمی ریاضی ای است که قادر به پیشگویی و بازتولید نتایج تجربی باشند.


$F = ma$
در معادلات
دقت
تجربه
محک دقت و صدق
F, m, a
در کمیتهای مورد استفاده در معادلات
$m = \frac{F}{a}$
غیر مستقیم با معادلات
روش تعیین دقیق کمیتها
m
مستقیم به طور تجربی
روش مستقیم مناسبتر است
زیرا مستقل است و به کمیتهای دیگر وابسته نیست و لذا نیاز به رجوع مداوم و خسته کننده به کمیتهای پایه را از بین می برد.
اما در عمل بین نتایج تجربی و نظری اختلاف وجود دارد
تلاش پژوهشگران برای رفع اختلاف
نسبیت و کوانتوم
تولد نظریات جدیدتر و کاملتر فیزیکی