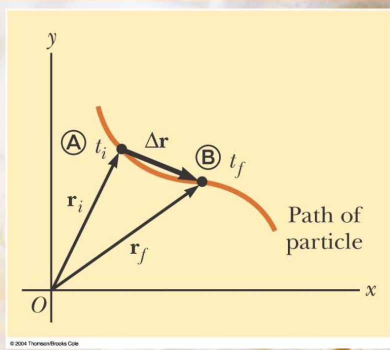
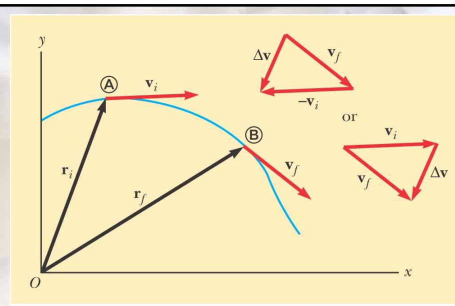
حرکت در دو بعد



$x \rightarrow \vec{r}$ مکان

$\Delta x \rightarrow \Delta\vec{r}$ جابه جایی



$v \rightarrow \vec{v}$ سرعت

$$v = \frac{\Delta x}{\Delta t} \qquad \vec{v} = \frac{\Delta \vec{r}}{\Delta t}$$

$$v = \frac{dx}{dt} \rightarrow \vec{v} = \frac{d\vec{r}}{dt}$$

a → $\vec{a}$ شتاب

$a = \frac{\Delta v}{\Delta t}$ → $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

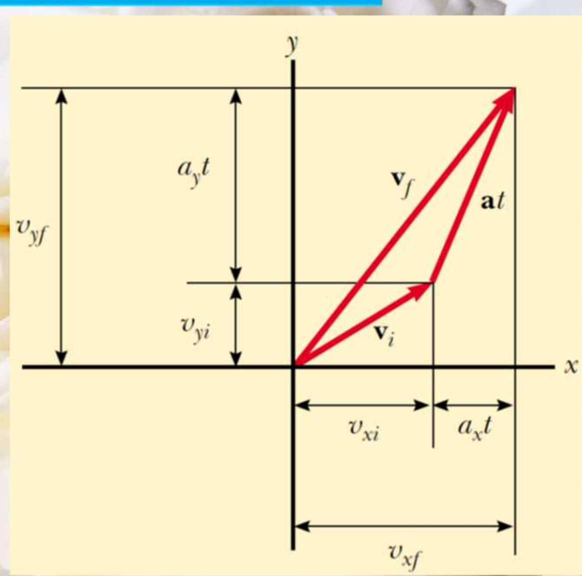
$v = \frac{dx}{dt}$ → $\vec{v} = \frac{d\vec{r}}{dt}$

شتاب ثابت در دو بعد

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$v_{xf} = v_{xi} + a_x t$$

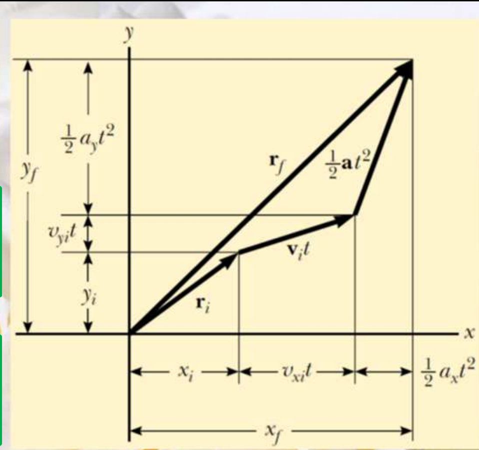
$$v_{yf} = v_{yi} + a_y t$$



$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

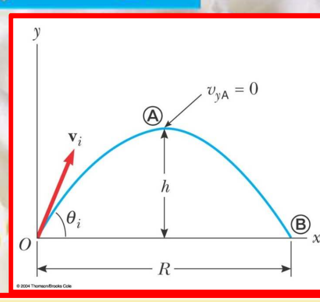
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

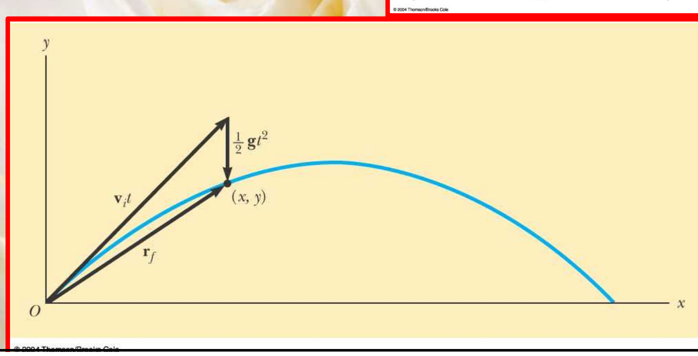
$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

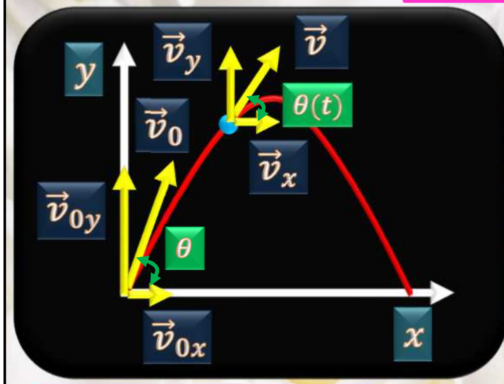


حرکت پرتابی

$$\mathbf{r} = \mathbf{r}_i + \mathbf{v}_i t - \tfrac{1}{2}\mathbf{g}t^2$$







$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$

$$v_x = v_{0x} = v_0 cos\theta$$

$$v_y = -gt + v_{0y}$$

$$v_y = -gt + v_0 sin\theta$$

$$x = v_0 cos\theta t$$

$$y = -\frac{1}{2}gt^2 + v_0 sin\theta t$$

$$\vec{v}_0 = v_{0x}\hat{\imath} + v_{0y}\hat{\jmath}$$

$$v_{0x} = v_0 cos\theta$$

$$v_{0y} = v_0 sin\theta$$

در نقطه اوج مولفه قایم سرعت صفر است، پس

$$v_y = -gt + v_0 sin\theta = 0$$

زمان لازم برای اثابت به زمین 2 برابر زمان لازم برای رسیدن به نقطه اوج است، پس

$$t_{Top} = \frac{v_0 sin\theta}{g}$$

$$t_{Earth} = \frac{2v_0 sin\theta}{g}$$

ارتفاع اوج با زمان اوج قابل محاسبه است:

$$y(t_{Top}) = H = -\frac{1}{2}g\left(\frac{v_0 sin\theta}{g}\right)^2 + v_0 sin\theta\frac{v_0 sin\theta}{g}$$

$$H = -\frac{1}{2}\frac{v_0^2 sin^2\theta}{g} + \frac{v_0^2 sin^2\theta}{g}$$

$$H = \frac{v_0^2 sin^2\theta}{2g}$$

برد پرتابه با زمان اثابت به زمین قابل محاسبه است

$$x(t_{Earth}) = R = v_0 cos\theta \frac{2v_0 sin\theta}{g}$$

$$R = \frac{v_0^2 sin2\theta}{g}$$

معادله حرکت با حذف زمان به صورت زیر به دست می آید

$$x = v_0 cos\theta t$$

$$t = \frac{x}{v_0 cos\theta}$$

$$y = -\frac{1}{2} g \left(\frac{x}{v_0 cos\theta}\right)^2 + v_0 sin\theta \frac{x}{v_0 cos\theta}$$

$$y = -\frac{g}{2v_0^2 cos^2\theta} x^2 + tan\theta x$$

برد پرتابه را می توان با معادله حرکت هم به دست آورد:

$$y = -\frac{g}{2v_0^2 cos^2\theta} x^2 + tan\theta x = 0$$

$$\left(-\frac{g}{2v_0^2 cos^2\theta} x + tan\theta\right) x = 0$$

$$x = 0$$

$$-\frac{g}{2v_0^2 cos^2\theta} x + tan\theta = 0$$

که همان برد پرتابه است

$$x = 2tan\theta v_0^2 cos^2\theta / g$$

$$x = 2sin\theta v_0^2 cos\theta / g$$

$$x = \frac{v_0^2 sin2\theta}{g}$$

ارتفاع اوج پرتابه را می توان با معادله حرکت هم به دست آورد:

$$\frac{dy}{dx} = -\frac{g}{v_0^2 cos^2\theta}x + tan\theta = 0$$

$$\frac{g}{v_0^2 cos^2\theta}x = tan\theta$$

$$x = tan\theta \frac{v_0^2 cos^2\theta}{g}$$

$$x = \frac{v_0^2 sin\theta cos\theta}{g}$$

که همان نصف برد است

$$y = -\frac{g}{2v_0^2 cos^2\theta}\left(\frac{v_0^2 sin\theta cos\theta}{g}\right)^2 + tan\theta\left(\frac{v_0^2 sin\theta cos\theta}{g}\right)$$

$$y = -\frac{v_0^2 sin^2\theta}{2g} + \frac{v_0^2 sin^2\theta}{g}$$

$$y = \frac{v_0^2 sin^2\theta}{2g}$$

که همان ارتفاع اوج پرتابه است

یک فرمول مفید بین H و R و θ

$$H = \frac{v_0^2 sin^2\theta}{2g}$$

$$R = \frac{v_0^2 sin2\theta}{g}$$

$$\frac{H}{R} = \frac{v_0^2 sin^2\theta}{2g}\frac{g}{v_0^2 sin2\theta}$$

$$\frac{H}{R} = \frac{tan\theta}{4}$$

$$\frac{4H}{R} = tan\theta$$

با این رابطه می توان زاویه اولیه پرتابه را حساب کرد

برد پرتابه تحت زاویه 45 درجه بیشینه می شود

$$R = \frac{v_0^2 sin2\theta}{g}$$

$$\frac{dR}{d\theta} = \frac{2v_0^2 cos2\theta}{g} = 0$$

$$cos2\theta = 0 = cos\frac{\pi}{2}$$

$$2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4}$$

زوایای متمم دارای بردهای یکسانی هستند

دو زاویه متمم

$$\theta + \gamma = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2} - \gamma$$

$$R = \frac{v_0^2 sin2\theta}{g}$$

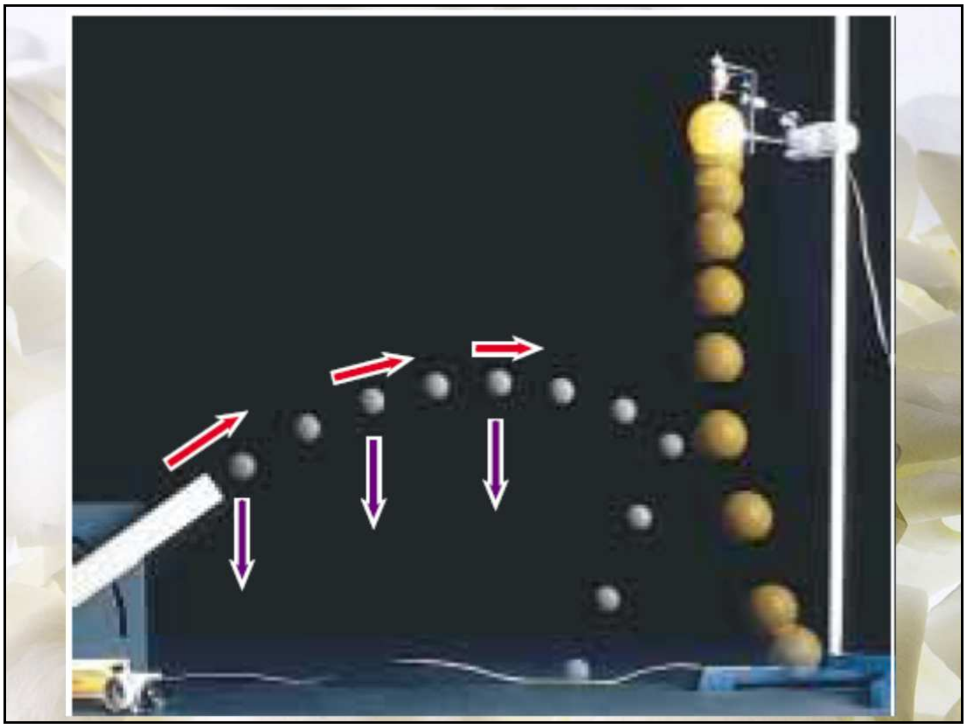
$$R = \frac{v_0^2 sin2\left(\frac{\pi}{2} - \gamma\right)}{g}$$

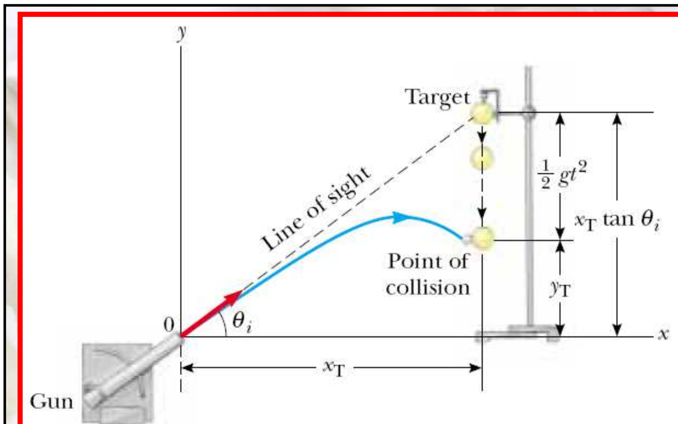
$$R = \frac{v_0^2 sin(\pi - 2\gamma)}{g}$$

$$R = \frac{v_0^2 sin2\gamma}{g}$$

Example 4.4 A Bull's-Eye Every Time

In a popular lecture demonstration, a projectile is fired at a target T in such a way that the projectile leaves the gun at the same time the target is dropped from rest, as shown in Figure 4.13. Show that if the gun is initially aimed at the stationary target, the projectile hits the target.





برخورد یعنی این که اگر

$$x_P = x_T$$

آنگاه

$$y_P = y_T$$

که این امر روی می دهد زیرا در صورت تحقق فرض، برقراری حکم بدیهی است:

$$y_T = x_T tan\theta_i - \frac{1}{2}gt^2$$

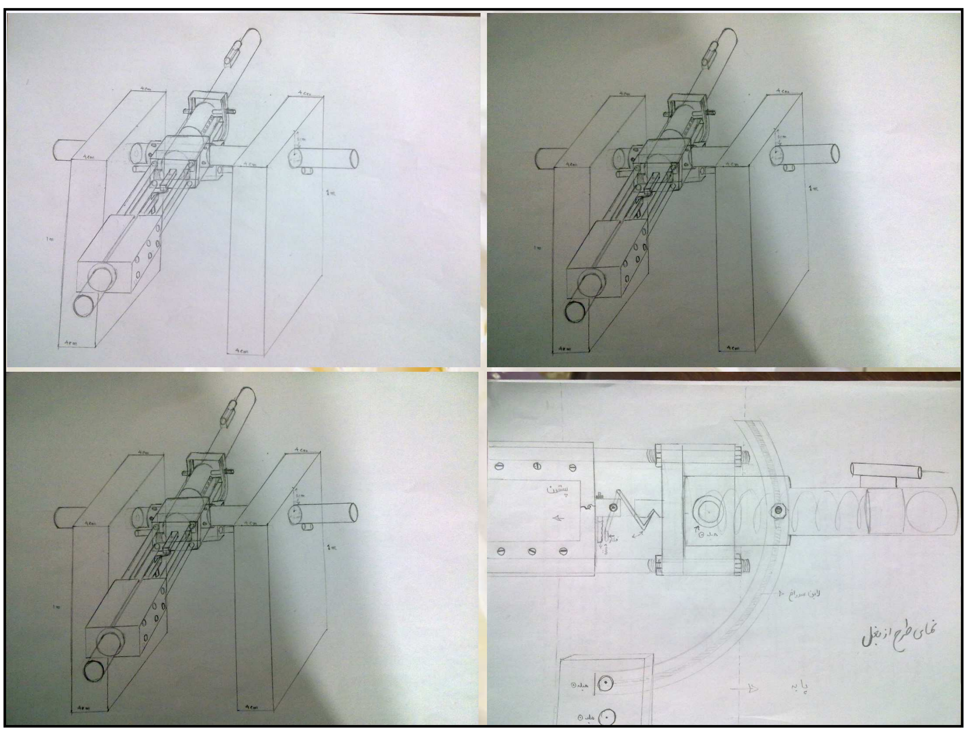
$$y_P = x_P tan\theta_i - \frac{1}{2}gt^2$$

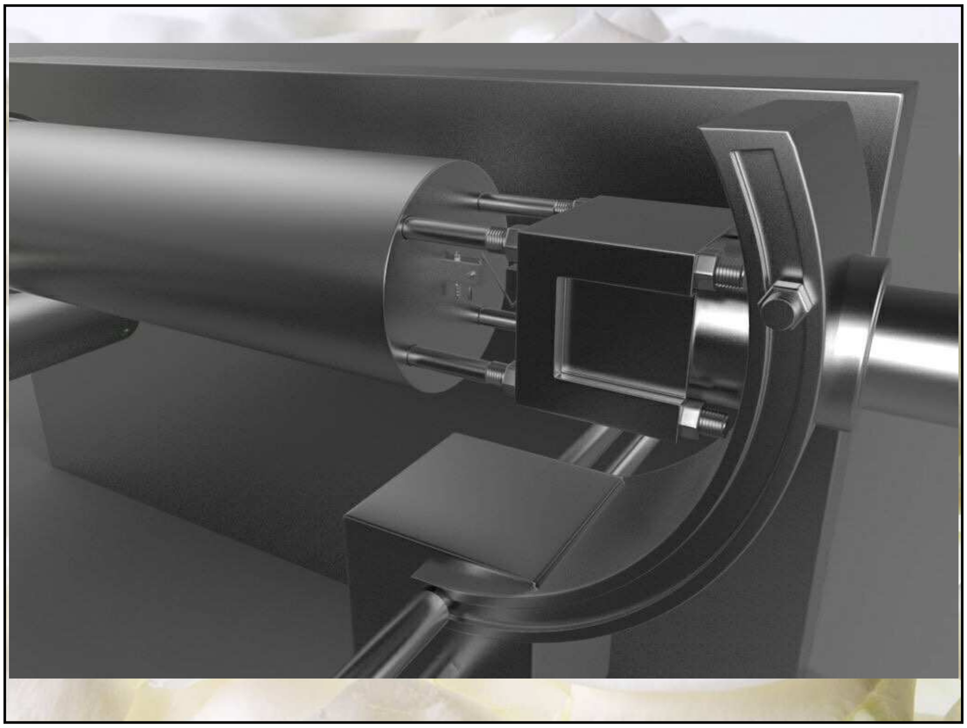
$$y_P = x_T tan\theta_i - \frac{1}{2}gt^2 = y_T$$

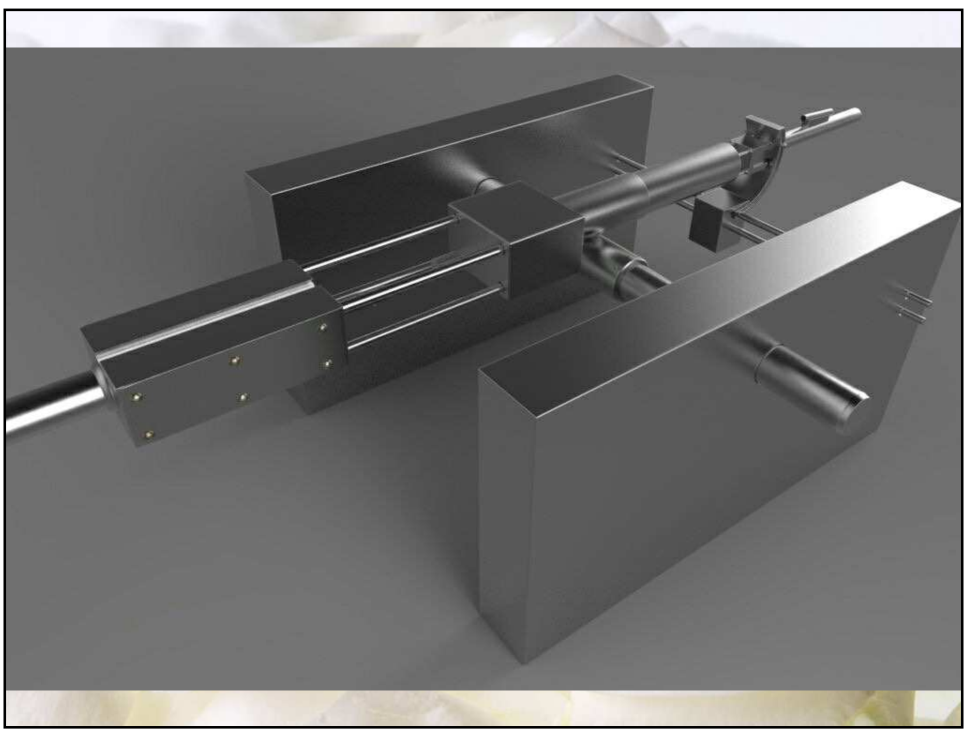
```
screen 12
cls
line(50,20)-(50,400),14
locate 2,4
print" y"
line(50,400)-(450,400),14
locate 27,57
print"x"
line(50,15)-(54,25),14
line(50,15)-(46,25),14
line(455,400)-(445,405),14
line(455,400)-(445,395),14
line(430,400)-(430,50),14
line(420,400)-(420,50),14
line(430,50)-(420,50),14
for x=420 to 430
   line(x,400)-(x,50) ,12
next x
circle(425,80),10,
line(416,77)-(370,77)
line(416,83)-(370,83)
line(370,77)-(370,83)
circle(370,94),10
```

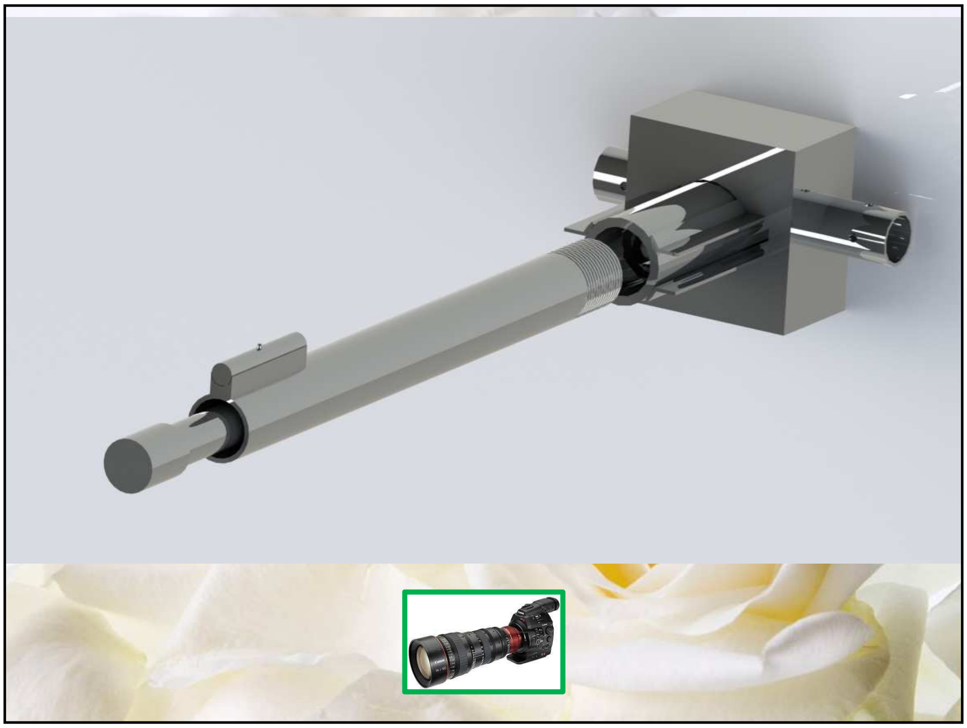
```
xt = 370
tetai = atn((400 - 94) / (370 - 53))
g = 9.8
v0 = 1.5 * sqr(317 * g / sin(2 * tetai))
rem print 53*tan(tetai)
s = 0.00001
for t=0 to 7.7723 step s
   YT = xt * tan(tetai) + 1 / 2 * g * T ^ 2 + 94 - xt * tan(tetai)
   circle(xt,yt),10,10
   ss = ss + s
   circle(xt,yt-.15*ss),10, 0
   circle(xt,yt), 10,10
   rem   circle(53,397),3
   xp = v0 * cos(tetai) * T + 53
   yp = -xp * tan(tetai) + 1 / 2 * g * T ^ 2 + 397 + 53 * tan(tetai)
   circle(xp,yp),2, 14

next t
```



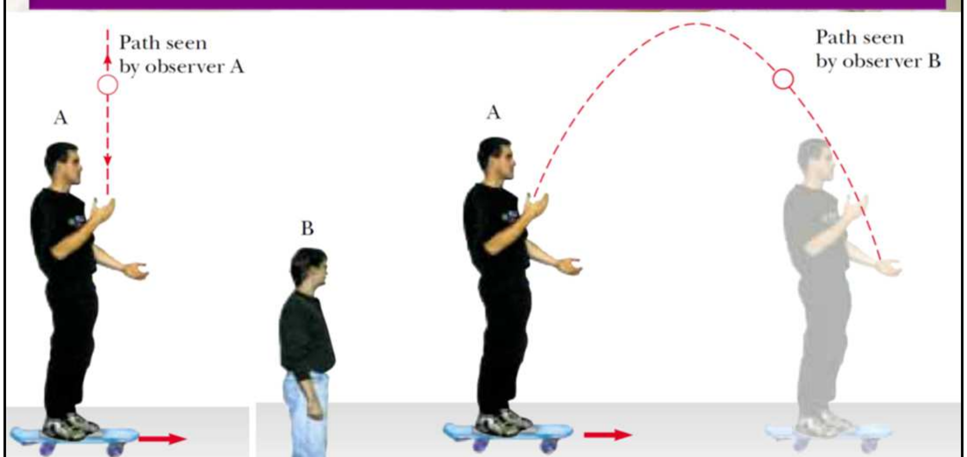






نسبیت کلاسیک یا نسبیت گالیله ای

نسبیت کلاسیک وقتی روی می دهد که دو ناظر نسبت به هم با سرعت ثابت در حرکت باشند



ناظر A حرکت را عمودی ولی ناظر B حرکت را به صورت پرتابی می بینند

$$\vec{r}' = \vec{r} - \vec{v}_0 t$$

$$\vec{v}' = \frac{d\vec{r}'}{dt}$$

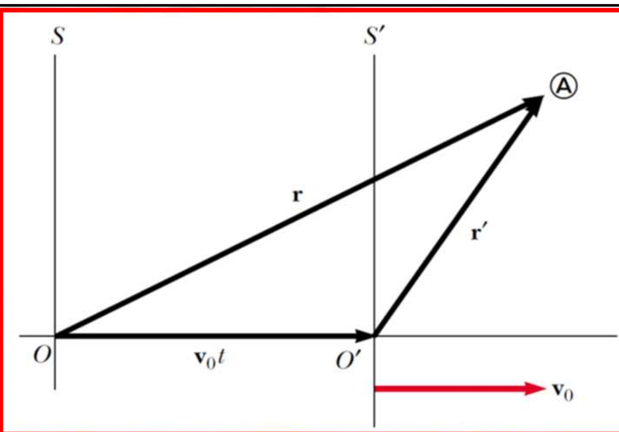
$$\vec{v}' = \frac{d\vec{r}}{dt} - \vec{v}_0$$

$$\vec{v}' = \vec{v} - \vec{v}_0$$

$$\vec{a}' = \frac{d\vec{v}'}{dt}$$

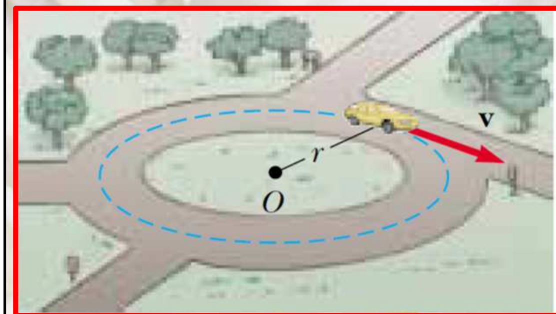
$$\vec{a}' = \frac{d\vec{v}}{dt} - 0 = \frac{d\vec{v}}{dt}$$

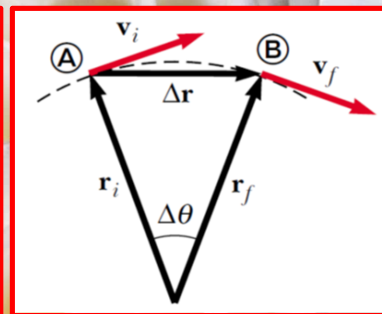
$$\boxed{\vec{a}' = \vec{a}}$$



بنابراین قوانین فیزیک در تمام دستگاه
های لخت یکسانند

حرکت دایروی یکنواخت

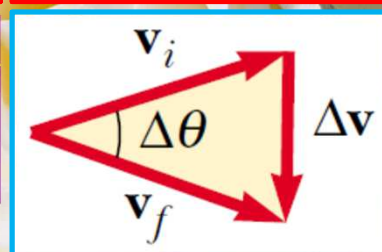




آیا تندی ثابت سرعت ثابت را ایجاب می کند؟

خیر، بردار سرعت می تواند دوران دایروی یکنواخت انجام دهد به طوری که در زوایای کوچک داشته باشیم:

$$\Delta\vec{v} \perp \vec{v}$$



توجیه شتاب جانب مرکز به طور هندسی

$$\Delta\theta = \frac{|\Delta\vec{r}|}{r}$$

$$\Delta\theta = \frac{|\Delta\vec{v}|}{v}$$

$$a = |\vec{a}| = \frac{|\Delta\vec{v}|}{\Delta t}$$

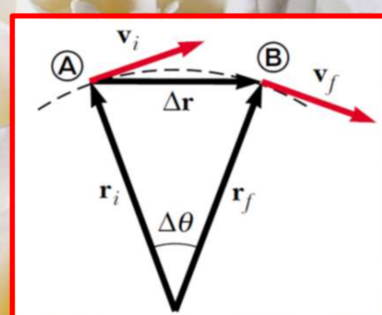
$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

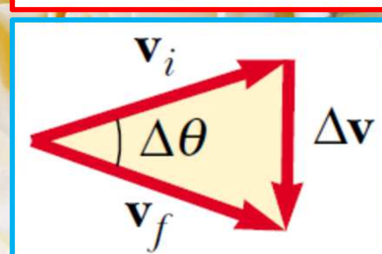
$$a = v\frac{\frac{|\Delta\vec{r}|}{r}}{\Delta t}$$

$$a = v\frac{\Delta\theta}{\Delta t}$$

$$\boxed{a = \frac{v^2}{r}}$$

$$a = \frac{v}{r}\frac{|\Delta\vec{r}|}{\Delta t}$$

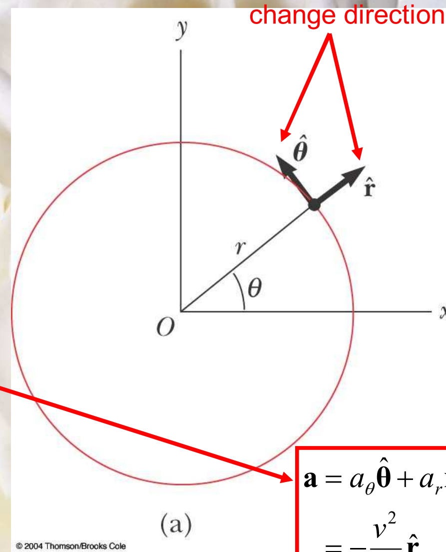




Uniform Circular Motion

- Note that **a** is *not* constant, but its magnitude is
- Can pick appropriate coordinate system and unit vectors for the problem
  - In polar coordinates *r* and $d\theta/dt$ are constant
  - Break acceleration down into tangential and radial components
  - Tangential component is zero for uniform circular motion

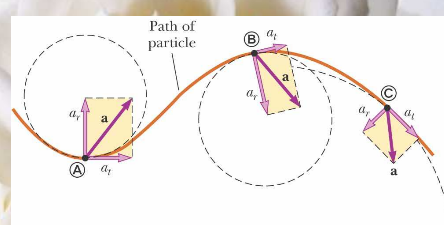
Unit vectors which change direction!



$$\mathbf{a} = a_\theta \hat{\boldsymbol{\theta}} + a_r \hat{\mathbf{r}} = -\frac{v^2}{r}\hat{\mathbf{r}}$$

Tangential & Radial Acceleration

- Can use tangential & radial acceleration even when not on circle
- Can also have non-zero tangential acceleration with circular motion
  - Rocket-on-a-string
  - Radial (centripetal) acceleration must increase in magnitude



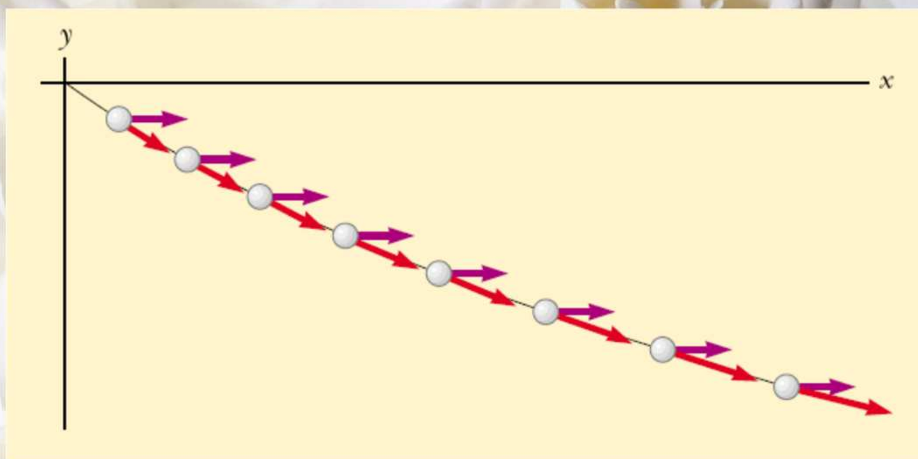
Quick Quiz 4.1 Which of the following cannot *possibly* be accelerating? (a) An object moving with a constant speed (b) An object moving with a constant velocity (c) An object moving along a curve.

Quick Quiz 4.2 Consider the following controls in an automobile: gas pedal, brake, steering wheel. The controls in this list that cause an acceleration of the car are (a) all three controls (b) the gas pedal and the brake (c) only the brake (d) only the gas pedal.

Example 4.1 Motion in a Plane

A particle starts from the origin at $t = 0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle moves in the xy plane with an x component of acceleration only, given by $a_x = 4.0\ \text{m/s}^2$.

(A) Determine the components of the velocity vector at any time and the total velocity vector at any time.



Solution After carefully reading the problem, we *conceptualize* what is happening to the particle. The components of the initial velocity tell us that the particle starts by moving toward the right and downward. The x component of velocity starts at 20 m/s and increases by 4.0 m/s every second. The y component of velocity never changes from its initial value of -15 m/s. We sketch a rough motion diagram of the situation in Figure 4.6. Because the particle is accelerating in the $+x$ direction, its velocity component in this direction will increase, so that the path will curve as shown in the diagram. Note that the spacing between successive images increases as time goes on because the speed is increasing. The placement of the acceleration and velocity vectors in Figure 4.6 helps us to further conceptualize the situation.

Because the acceleration is constant, we *categorize* this problem as one involving a particle moving in two dimensions with constant acceleration. To *analyze* such a problem, we use the equations developed in this section. To begin the mathematical analysis, we set $v_{xi} = 20\ \text{m/s}$, $v_{yi} = -15\ \text{m/s}$, $a_x = 4.0\ \text{m/s}^2$, and $a_y = 0$.

Equations 4.8a give

(1) $$v_{xf} = v_{xi} + a_x t = (20 + 4.0t)\ \text{m/s}$$

(2) $$v_{yf} = v_{yi} + a_y t = -15\ \text{m/s} + 0 = -15\ \text{m/s}$$

Therefore

$$\mathbf{v}_f = v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}} = [(20 + 4.0t)\hat{\mathbf{i}} - 15\hat{\mathbf{j}}]\ \text{m/s}$$

We could also obtain this result using Equation 4.8 directly, noting that $\mathbf{a} = 4.0\hat{\mathbf{i}}\ \text{m/s}^2$ and $\mathbf{v}_i = [20\hat{\mathbf{i}} - 15\hat{\mathbf{j}}]\ \text{m/s}$. To *finalize* this part, notice that the x component of velocity increases in time while the y component remains constant; this is consistent with what we predicted.

(B) Calculate the velocity and speed of the particle at $t = 5.0$ s.

Solution With $t = 5.0$ s, the result from part (A) gives

$$\mathbf{v}_f = [(20 + 4.0(5.0))\hat{\mathbf{i}} - 15\hat{\mathbf{j}}]\ \text{m/s} = (40\hat{\mathbf{i}} - 15\hat{\mathbf{j}})\ \text{m/s}$$

This result tells us that at $t = 5.0$ s, $v_{xf} = 40$ m/s and $v_{yf} = -15$ m/s. Knowing these two components for this two-dimensional motion, we can find both the direction and the magnitude of the velocity vector. To determine the angle θ that $\mathbf{v}$ makes with the x axis at $t = 5.0$ s, we use the fact that $\tan\theta = v_{yf}/v_{xf}$:

$$\text{(3)} \qquad \theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-15\ \text{m/s}}{40\ \text{m/s}}\right) = -21^\circ$$

where the negative sign indicates an angle of 21° below the positive x axis. The speed is the magnitude of $\mathbf{v}_f$:

$$v_f = |\mathbf{v}_f| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(40)^2 + (-15)^2}\ \text{m/s} = 43\ \text{m/s}$$

To *finalize* this part, we notice that if we calculate v_i from the x and y components of $\mathbf{v}_i$, we find that $v_f > v_i$. Is this consistent with our prediction?

(C) Determine the x and y coordinates of the particle at any time t and the position vector at this time.

Solution Because $x_i = y_i = 0$ at $t = 0$, Equation 4.9a gives

$$x_f = v_{xi}t + \tfrac{1}{2}a_x t^2 = \boxed{(20t + 2.0t^2)\ \text{m}}$$

$$y_f = v_{yi}t = \boxed{(-15t)\ \text{m}}$$

Therefore, the position vector at any time t is

$$(4) \qquad \mathbf{r}_f = x_f\hat{\mathbf{i}} + y_f\hat{\mathbf{j}} = \boxed{[(20t + 2.0t^2)\hat{\mathbf{i}} - 15t\hat{\mathbf{j}}]\ \text{m}}$$

(Alternatively, we could obtain $\mathbf{r}_f$ by applying Equation 4.9 directly, with $\mathbf{v}_f = (20\hat{\mathbf{i}} - 15\hat{\mathbf{j}})$ m/s and $\mathbf{a} = 4.0\hat{\mathbf{i}}$ m/s^2. Try it!) Thus, for example, at $t = 5.0$ s, $x = 150$ m, $y = -75$ m, and $\mathbf{r}_f = (150\hat{\mathbf{i}} - 75\hat{\mathbf{j}})$ m. The magnitude of the displacement of the particle from the origin at $t = 5.0$ s is the magnitude of $\mathbf{r}_f$ at this time:

$$r_f = |\mathbf{r}_f| = \sqrt{(150)^2 + (-75)^2}\ \text{m} = 170\ \text{m}$$

Note that this is *not* the distance that the particle travels in this time! Can you determine this distance from the available data?

To *finalize* this problem, let us consider a limiting case for very large values of t in the following **What If?**

What If? What if we wait a very long time and then observe the motion of the particle? How would we describe the motion of the particle for large values of the time?

Answer Looking at Figure 4.6, we see the path of the particle curving toward the x axis. There is no reason to assume that this tendency will change, so this suggests that the path will become more and more parallel to the x axis as time grows large. Mathematically, let us consider Equations (1) and (2). These show that the y component of the velocity remains constant while the x component grows linearly with t. Thus, when t is very large, the x component of the velocity will be much larger than the y component, suggesting that the velocity vector becomes more and more parallel to the x axis.

Equation (3) gives the angle that the velocity vector makes with the x axis. Notice that $\theta \rightarrow 0$ as the denominator (v_{xf}) becomes much larger than the numerator (v_{yf}).

Despite the fact that the velocity vector becomes more and more parallel to the x axis, the particle does not approach a limiting value of y. Equation (4) shows that both x_f and y_f continue to grow with time, although x_f grows much faster.

Quick Quiz 4.3 Suppose you are running at constant velocity and you wish to throw a ball such that you will catch it as it comes back down. In what direction should you throw the ball relative to you? (a) straight up (b) at an angle to the ground that depends on your running speed (c) in the forward direction.

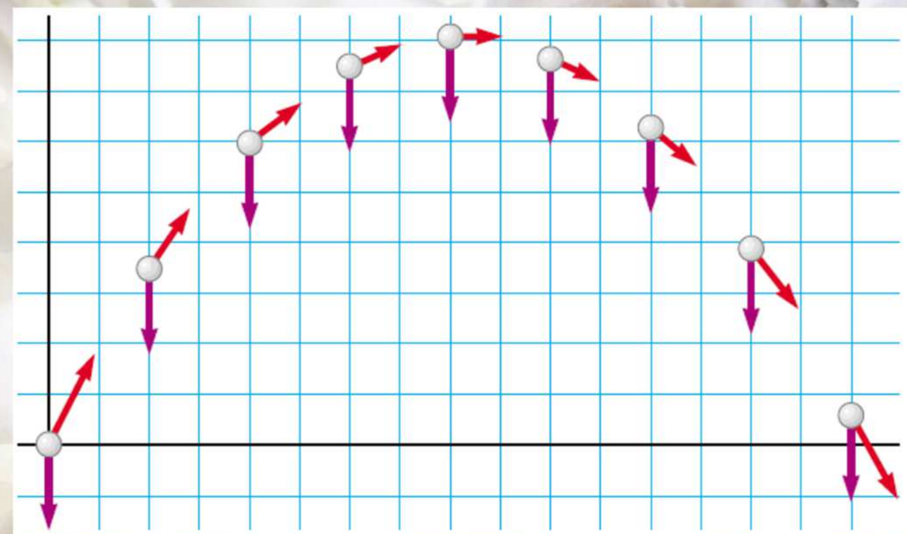
Quick Quiz 4.4 As a projectile thrown upward moves in its parabolic path (such as in Figure 4.8), at what point along its path are the velocity and acceleration vectors for the projectile perpendicular to each other? (a) nowhere (b) the highest point (c) the launch point.

Quick Quiz 4.5 As the projectile in Quick Quiz 4.4 moves along its path, at what point are the velocity and acceleration vectors for the projectile parallel to each other? (a) nowhere (b) the highest point (c) the launch point.

Example 4.2 Approximating Projectile Motion

A ball is thrown in such a way that its initial vertical and horizontal components of velocity are 40 m/s and 20 m/s, respectively. Estimate the total time of flight and the distance the ball is from its starting point when it lands.

Solution A motion diagram like Figure 4.9 helps us *conceptualize* the problem. The phrase "A ball is thrown" allows us to *categorize* this as a projectile motion problem, which we *analyze* by continuing to study Figure 4.9. The acceleration vectors are all the same, pointing downward with a magnitude of nearly 10 m/s^2. The velocity vectors change direction. Their horizontal components are all the same: 20 m/s.

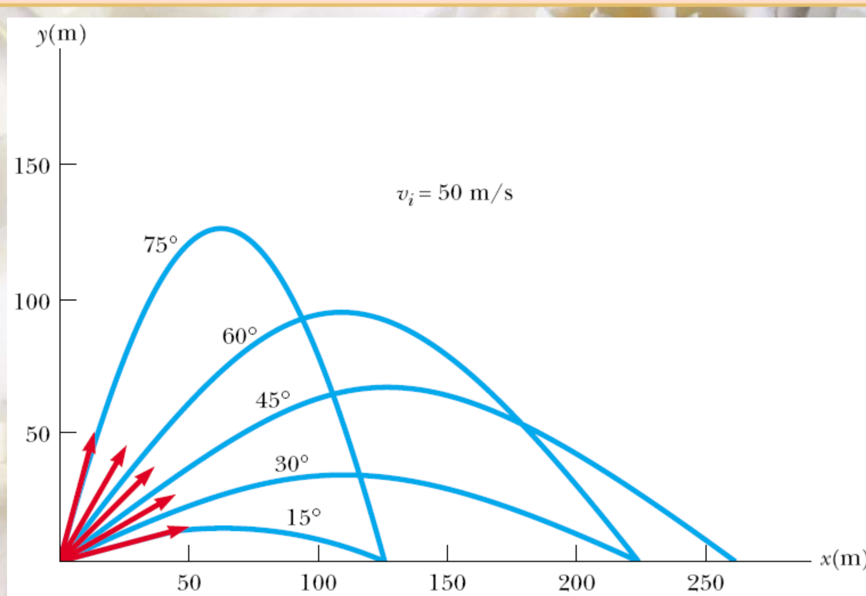


Remember that the two velocity components are independent of each other. By considering the vertical motion first, we can determine how long the ball remains in the air. Because the vertical motion is free-fall, the vertical components of the velocity vectors change, second by second, from 40 m/s to roughly 30, 20, and 10 m/s in the upward direction, and then to 0 m/s. Subsequently, its velocity becomes 10, 20, 30, and 40 m/s in the downward direction. Thus it takes the ball about 4 s to go up and another 4 s to come back down, for a total time of flight of approximately 8 s.

Now we shift our analysis to the horizontal motion. Because the horizontal component of velocity is 20 m/s, and because the ball travels at this speed for 8 s, it ends up approximately 160 m from its starting point.

This is the first example that we have performed for projectile motion. In subsequent projectile motion problems, keep in mind the importance of separating the two components and of making approximations to give you rough expected results.

Quick Quiz 4.6 Rank the launch angles for the five paths in Figure 4.11 with respect to time of flight, from the shortest time of flight to the longest.



PROBLEM-SOLVING HINTS

Projectile Motion

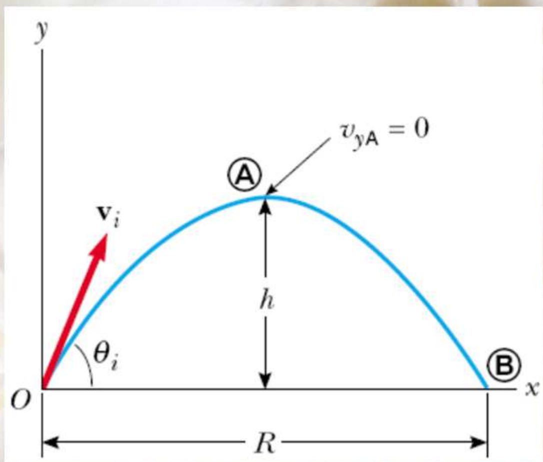
We suggest that you use the following approach to solving projectile motion problems:

- Select a coordinate system and resolve the initial velocity vector into x and y components.
- Follow the techniques for solving constant-velocity problems to analyze the horizontal motion. Follow the techniques for solving constant-acceleration problems to analyze the vertical motion. The x and y motions share the same time t.

Example 4.3 The Long Jump

A long-jumper (Fig. 4.12) leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.

(A) How far does he jump in the horizontal direction? (Assume his motion is equivalent to that of a particle.)



Solution We *conceptualize* the motion of the long-jumper as equivalent to that of a simple projectile such as the ball in Example 4.2, and *categorize* this problem as a projectile motion problem. Because the initial speed and launch angle are given, and because the final height is the same as the initial height, we further categorize this problem as satisfying the conditions for which Equations 4.13 and 4.14 can be used. This is the most direct way to *analyze* this problem, although the general methods that we have been describing will always give the correct answer. We will take the general approach and use components. Figure 4.10 provides a graphical representation of the flight of the long-jumper.

As before, we set our origin of coordinates at the takeoff point and label the peak as A and the landing point as B. The horizontal motion is described by Equation 4.11:

$$x_f = x_B = (v_i \cos \theta_i) t_B = (11.0 \text{ m/s})(\cos 20.0°) t_B$$

The value of x_B can be found if the time of landing t_B is known. We can find t_B by remembering that $a_y = -g$ and by using the y part of Equation 4.8a. We also note that at the top of the jump the vertical component of velocity v_{yA} is zero:

$$v_{yf} = v_{yA} = v_i \sin \theta_i - g t_A$$

$$0 = (11.0 \text{ m/s}) \sin 20.0° - (9.80 \text{ m/s}^2) t_A$$

$$t_A = 0.384 \text{ s}$$

This is the time at which the long-jumper is at the *top* of the jump. Because of the symmetry of the vertical motion, another 0.384 s passes before the jumper returns to the ground. Therefore, the time at which the jumper lands is $t_B=2t_A=0.768$ s. Substituting this value into the above expression for x_f gives

$$x_f = x_B = (11.0\text{ m/s})(\cos 20.0°)(0.768\text{ s}) = \boxed{7.94\text{ m}}$$

This is a reasonable distance for a world-class athlete.

(B) What is the maximum height reached?

Solution We find the maximum height reached by using Equation 4.12:

$$\begin{aligned} y_{max} = y_A &= (v_i \sin\theta_i)\,t_A - \tfrac{1}{2}g{t_A}^2 \\ &= (11.0\text{ m/s})(\sin 20.0°)(0.384\text{ s}) \\ &\qquad -\tfrac{1}{2}(9.80\text{ m/s}^2)(0.384\text{ s})^2 = \boxed{0.722\text{ m}} \end{aligned}$$

To *finalize* this problem, find the answers to parts (A) and (B) using Equations 4.13 and 4.14. The results should agree. Treating the long-jumper as a particle is an oversimplification. Nevertheless, the values obtained are consistent with experience in sports. We learn that we can model a complicated system such as a long-jumper as a particle and still obtain results that are reasonable.

Example 4.5 That's Quite an Arm!

A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal with an initial speed of 20.0 m/s, as shown in Figure 4.14. If the height of the building is 45.0 m,

(A) how long does it take the stone to reach the ground?

Solution We *conceptualize* the problem by studying Figure 4.14, in which we have indicated the various parameters. By now, it should be natural to *categorize* this as a projectile motion problem.

To *analyze* the problem, let us once again separate motion into two components. The initial x and y components of the stone's velocity are

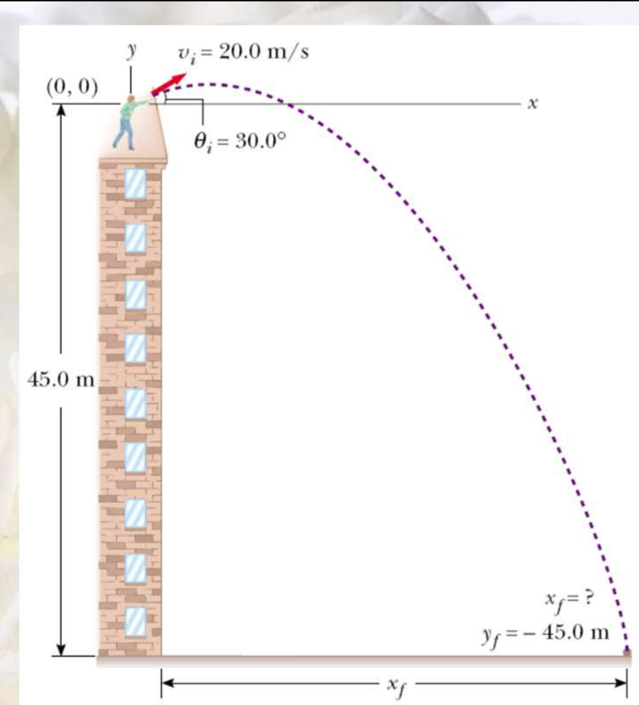
$$v_{xi} = v_i \cos\theta_i = (20.0\ \text{m/s})\cos 30.0° = 17.3\ \text{m/s}$$

$$v_{yi} = v_i \sin\theta_i = (20.0\ \text{m/s})\sin 30.0° = 10.0\ \text{m/s}$$

To find t, we can use $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$(Eq. 4.9a) with $y_i = 0$, $y_f = -45.0$ m, $a_y = -g$, and $v_{yi} = 10.0$ m/s (there is a negative sign on the numerical value of y_f because we have chosen the top of the building as the origin):

$$-45.0\ \text{m} = (10.0\ \text{m/s})t - \tfrac{1}{2}(9.80\ \text{m/s}^2)t^2$$

Solving the quadratic equation for t gives, for the positive root, $t =$ 4.22 s. To *finalize* this part, think: Does the negative root have any physical meaning?



(B) What is the speed of the stone just before it strikes the ground?

Solution We can use Equation 4.8a, $v_{yf} = v_{yi} + a_y t$, with $t = 4.22$ s to obtain the y component of the velocity just before the stone strikes the ground:

$$v_{yf} = 10.0 \text{ m/s} - (9.80 \text{ m/s}^2)(4.22 \text{ s}) = -31.4 \text{ m/s}$$

Because $v_{xf} = v_{xi} = 17.3$ m/s, the required speed is

$$v_f = \sqrt{v_{xf}{}^2 + v_{yf}{}^2} = \sqrt{(17.3)^2 + (-31.4)^2} \text{ m/s} = \boxed{35.9 \text{ m/s}}$$

To *finalize* this part, is it reasonable that the y component of the final velocity is negative? Is it reasonable that the final speed is larger than the initial speed of 20.0 m/s?

What If? What if a horizontal wind is blowing in the same direction as the ball is thrown and it causes the ball to have a horizontal acceleration component $a_x = 0.500\text{ m/s}^2$. Which part of this example, (A) or (B), will have a different answer?

Answer Recall that the motions in the x and y directions are independent. Thus, the horizontal wind cannot affect the vertical motion. The vertical motion determines the time of the projectile in the air, so the answer to (A) does not change. The wind will cause the horizontal velocity component to increase with time, so that the final speed will change in part (B).

We can find the new final horizontal velocity component by using Equation 4.8a:

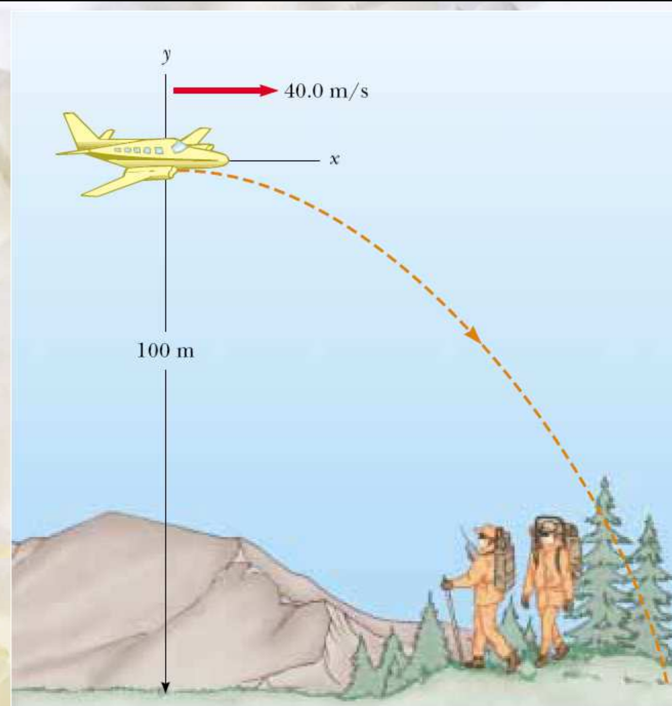
$$v_{xf} = v_{xi} + a_x t = 17.3\text{ m/s} + (0.500\text{ m/s}^2)(4.22\text{ s})$$
$$= 19.4\text{ m/s}$$

and the new final speed:

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(19.4)^2 + (-31.4)^2}\text{ m/s} = 36.9\text{ m/s}$$

Example 4.6 The Stranded Explorers

A plane drops a package of supplies to a party of explorers, as shown in Figure 4.15. If the plane is traveling horizontally at 40.0 m/s and is 100 m above the ground, where does the package strike the ground relative to the point at which it is released?



Solution *Conceptualize* what is happening with the assistance of Figure 4.15. The plane is traveling horizontally when it drops the package. Because the package is in free-fall while moving in the horizontal direction, we *categorize* this as a projectile motion problem. To *analyze* the problem, we choose the coordinate system shown in Figure 4.15, in which the origin is at the point of release of the package. Consider first its horizontal motion. The only equation available for finding the position along the horizontal direction is $x_f = x_i + v_{xi}t$ (Eq. 4.9a). The initial x component of the package velocity is the same as that of the plane when the package is released: 40.0 m/s. Thus, we have

$$x_f = (40.0 \text{ m/s})\, t$$

If we know t, the time at which the package strikes the ground, then we can determine x_f, the distance the package travels in the horizontal direction. To find t, we use the equations that describe the vertical motion of the package. We know that, at the instant the package hits the ground, its y coordinate is $y_f = -100$ m. We also know that the initial vertical component of the package velocity v_{yi} is zero because at the moment of release, the package has only a horizontal component of velocity.

From Equation 4.9a, we have

$$y_f = -\tfrac{1}{2}gt^2$$

$$-100 \text{ m} = -\tfrac{1}{2}(9.80 \text{ m/s}^2)\,t^2$$

$$t = 4.52 \text{ s}$$

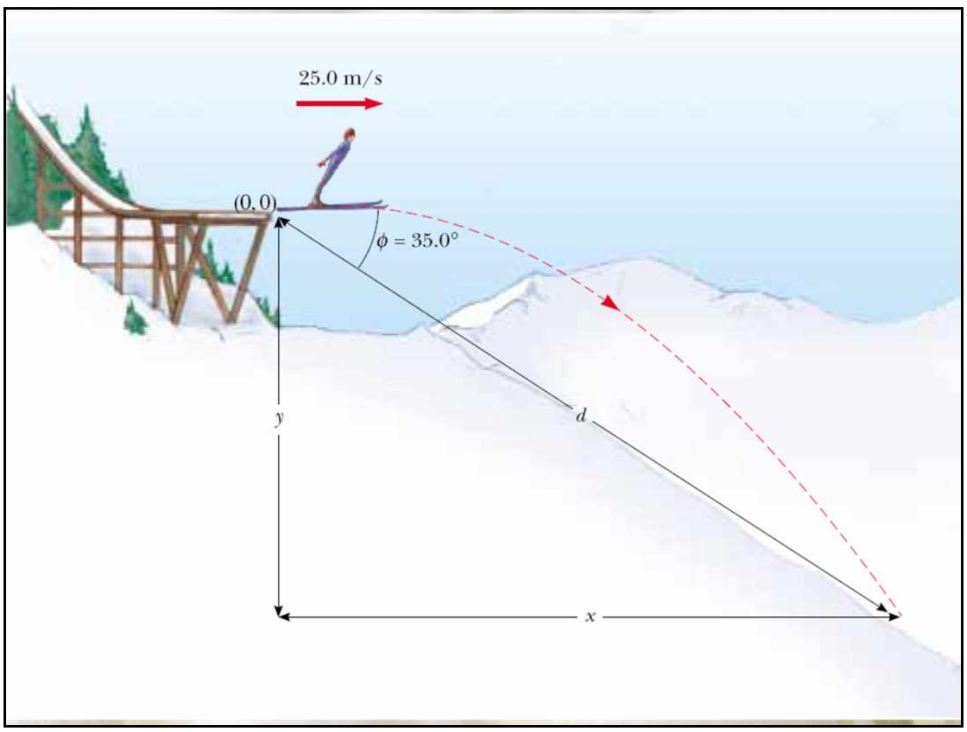
Substitution of this value for the time into the equation for the x coordinate gives

$$x_f = (40.0 \text{ m/s})(4.52 \text{ s}) = \boxed{181 \text{ m}}$$

The package hits the ground 181 m to the right of the drop point. To *finalize* this problem, we learn that an object dropped from a moving airplane does not fall straight down. It hits the ground at a point different from the one right below the plane where it was released. This was an important consideration for free-fall bombs such as those used in World War II.

Example 4.7 The End of the Ski Jump

A ski-jumper leaves the ski track moving in the horizontal direction with a speed of 25.0 m/s, as shown in Figure 4.16. The landing incline below him falls off with a slope of 35.0°. Where does he land on the incline?



Solution We can *conceptualize* this problem based on observations of winter Olympic ski competitions. We observe the skier to be airborne for perhaps 4 s and go a distance of about 100 m horizontally. We should expect the value of d, the distance traveled along the incline, to be of the same order of magnitude. We *categorize* the problem as that of a particle in projectile motion.

To *analyze* the problem, it is convenient to select the beginning of the jump as the origin. Because $v_{xi} = 25.0$ m/s and $v_{yi} = 0$, the x and y component forms of Equation 4.9a are

$$(1) \qquad x_f = v_{xi}t = (25.0 \text{ m/s})\, t$$

$$(2) \qquad y_f = v_{yi}t + \tfrac{1}{2}a_y t^2 = -\tfrac{1}{2}(9.80 \text{ m/s}^2)\, t^2$$

From the right triangle in Figure 4.16, we see that the jumper's x and y coordinates at the landing point are $x_f = d\cos 35.0°$ and $y_f = -d\sin 35.0°$. Substituting these relationships into (1) and (2), we obtain

$$(3) \qquad d\cos 35.0° = (25.0 \text{ m/s})\, t$$

$$(4) \qquad -d\sin 35.0° = -\tfrac{1}{2}(9.80 \text{ m/s}^2)\, t^2$$

Solving (3) for t and substituting the result into (4), we find that $d = 109$ m. Hence, the x and y coordinates of the point at which the skier lands are

$$x_f = d\cos 35.0° = (109 \text{ m})\cos 35.0° = \boxed{89.3 \text{ m}}$$

$$y_f = -d\sin 35.0° = -(109 \text{ m})\sin 35.0° = \boxed{-62.5 \text{ m}}$$

To *finalize* the problem, let us compare these results to our expectations. We expected the horizontal distance to be on the order of 100 m, and our result of 89.3 m is indeed on

What If? **Suppose everything in this example is the same except that the ski jump is curved so that the jumper is projected upward at an angle from the end of the track. Is this a better design in terms of maximizing the length of the jump?**

Answer If the initial velocity has an upward component, the skier will be in the air longer, and should therefore travel further. However, tilting the initial velocity vector upward will reduce the horizontal component of the initial velocity. Thus, angling the end of the ski track upward at a *large* angle may actually *reduce* the distance. Consider the extreme case. The skier is projected at 90° to the horizontal, and simply goes up and comes back down at the end of the ski track! This argument suggests that there must be an optimal angle between 0 and 90° that represents a balance between making the flight time longer and the horizontal velocity component smaller.

We can find this optimal angle mathematically. We modify equations (1) through (4) in the following way, assuming that the skier is projected at an angle θ with respect to the horizontal:

$$(1) \text{ and } (3) \quad \rightarrow \quad x_f = (v_i \cos\theta)t = d\cos\phi$$

$$(2) \text{ and } (4) \quad \rightarrow \quad y_f = (v_i \sin\theta)t - \tfrac{1}{2}gt^2 = -d\sin\phi$$

If we eliminate the time t between these equations and then use differentiation to maximize d in terms of θ, we arrive (after several steps—see Problem 72!) at the following equation for the angle θ that gives the maximum value of d:

$$\theta = 45° - \frac{\phi}{2}$$

For the slope angle in Figure 4.16, $\phi = 35.0°$; this equation results in an optimal launch angle of $\theta = 27.5°$. Notice that for a slope angle of $\phi = 0°$, which represents a horizontal plane, this equation gives an optimal launch angle of $\theta = 45°$, as we would expect (see Figure 4.11).

Quick Quiz 4.7 Which of the following correctly describes the centripetal acceleration vector for a particle moving in a circular path? (a) constant and always perpendicular to the velocity vector for the particle (b) constant and always parallel to the velocity vector for the particle (c) of constant magnitude and always perpendicular to the velocity vector for the particle (d) of constant magnitude and always parallel to the velocity vector for the particle.

Quick Quiz 4.8 A particle moves in a circular path of radius r with speed v. It then increases its speed to $2v$ while traveling along the same circular path. The centripetal acceleration of the particle has changed by a factor of (a) 0.25 (b) 0.5 (c) 2 (d) 4 (e) impossible to determine

Example 4.8 The Centripetal Acceleration of the Earth

What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun?

Solution We *conceptualize* this problem by bringing forth our familiar mental image of the Earth in a circular orbit around the Sun. We will simplify the problem by modeling the Earth as a particle and approximating the Earth's orbit as circular (it's actually slightly elliptical). This allows us to *categorize* this problem as that of a particle in uniform circular motion. When we begin to *analyze* this problem, we realize that we do not know the orbital speed of the Earth in Equation 4.15. With the help of Equation 4.16, however, we can recast Equation 4.15 in terms of the period of the Earth's orbit, which we know is one year:

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2(1.496 \times 10^{11}\,\mathrm{m})}{(1\,\mathrm{yr})^2}\left(\frac{1\,\mathrm{yr}}{3.156 \times 10^7\,\mathrm{s}}\right)^2$$

$$= \boxed{5.93 \times 10^{-3}\,\mathrm{m/s^2}}$$

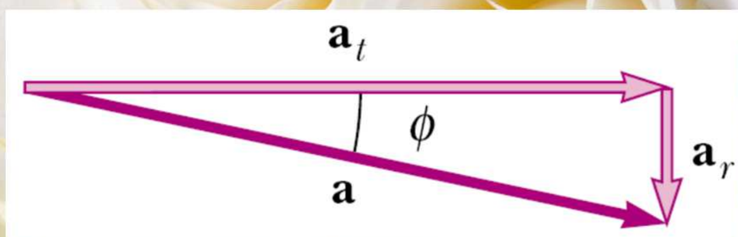
To *finalize* this problem, note that this acceleration is much smaller than the free-fall acceleration on the surface of the Earth. An important thing we learned here is the technique of replacing the speed v in terms of the period T of the motion.

Quick Quiz 4.9 A particle moves along a path and its speed increases with time. In which of the following cases are its acceleration and velocity vectors parallel? (a) the path is circular (b) the path is straight (c) the path is a parabola (d) never.

Quick Quiz 4.10 A particle moves along a path and its speed increases with time. In which of the following cases are its acceleration and velocity vectors perpendicular everywhere along the path? (a) the path is circular (b) the path is straight (c) the path is a parabola (d) never.

Example 4.9 Over the Rise

A car exhibits a constant acceleration of 0.300 m/s^2 parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like a circle of radius 500 m. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of 6.00 m/s. What is the direction of the total acceleration vector for the car at this instant?



Solution *Conceptualize* the situation using Figure 4.20a. Because the car is moving along a curved path, we can *categorize* this as a problem involving a particle experiencing both tangential and radial acceleration. Now we recognize that this is a relatively simple plug-in problem. The radial acceleration is given by Equation 4.19. With $v = 6.00\ \mathrm{m/s}$ and $r = 500\ \mathrm{m}$, we find that

$$a_r = -\frac{v^2}{r} = -\frac{(6.00\ \mathrm{m/s})^2}{500\ \mathrm{m}} = \boxed{-0.0720\ \mathrm{m/s^2}}$$

The radial acceleration vector is directed straight downward while the tangential acceleration vector has magnitude $0.300\ \mathrm{m/s^2}$ and is horizontal. Because $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, the magnitude of $\mathbf{a}$ is

$$a = \sqrt{a_r^{\,2} + a_t^{\,2}} = \sqrt{(-0.0720)^2 + (0.300)^2}\ \mathrm{m/s^2}$$

$$= \boxed{0.309\ \mathrm{m/s^2}}$$

If ϕ is the angle between $\mathbf{a}$ and the horizontal, then

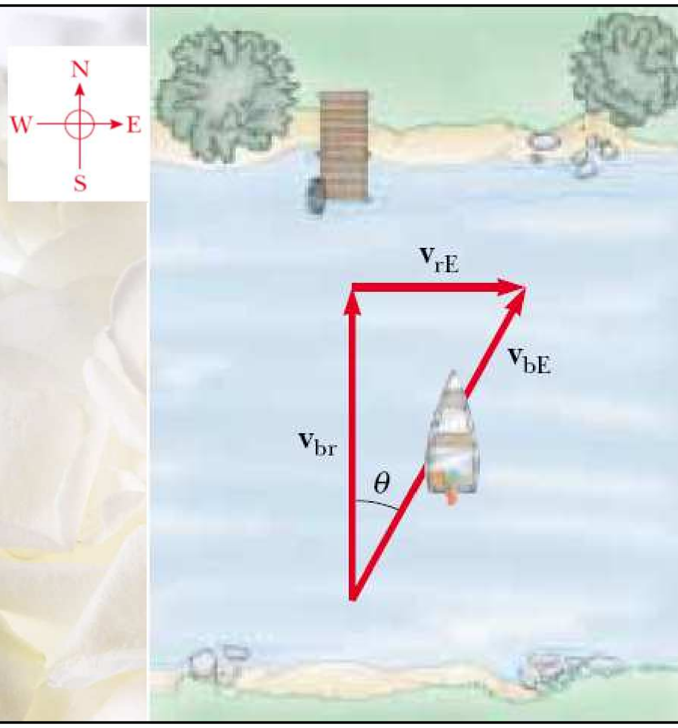
$$\phi = \tan^{-1}\frac{a_r}{a_t} = \tan^{-1}\left(\frac{-0.0720\ \mathrm{m/s^2}}{0.300\ \mathrm{m/s^2}}\right) = \boxed{-13.5^\circ}$$

This angle is measured downward from the horizontal. (See Figure 4.20b.)

Quick Quiz 4.11 A passenger, observer A, in a car traveling at a constant horizontal velocity of magnitude 60 mi/h pours a cup of coffee for the tired driver. Observer B stands on the side of the road and watches the pouring process through the window of the car as it passes. Which observer(s) sees a parabolic path for the coffee as it moves through the air? (a) A (b) B (c) both A and B (d) neither A nor B.

Example 4.10 A Boat Crossing a River

A boat heading due north crosses a wide river with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth. Determine the velocity of the boat relative to an observer standing on either bank.



Solution To *conceptualize* this problem, imagine moving across a river while the current pushes you along the river. You will not be able to move directly across the river, but will end up downstream, as suggested in Figure 4.24. Because of the separate velocities of you and the river, we can *categorize* this as a problem involving relative velocities. We will *analyze* this problem with the techniques discussed in this section. We know $\mathbf{v}_{br}$, the velocity of the *boat* relative to the *river*, and $\mathbf{v}_{rE}$, the velocity of the *river* relative to *Earth*. What we must find is $\mathbf{v}_{bE}$, the velocity of the *boat* relative to *Earth*. The relationship between these three quantities is

$$\mathbf{v}_{bE} = \mathbf{v}_{br} + \mathbf{v}_{rE}$$

The terms in the equation must be manipulated as vector quantities; the vectors are shown in Figure 4.24. The quantity $\mathbf{v}_{br}$ is due north, $\mathbf{v}_{rE}$ is due east, and the vector sum of the two, $\mathbf{v}_{bE}$, is at an angle θ, as defined in Figure 4.24. Thus, we can find the speed v_{bE} of the boat relative to Earth by using the Pythagorean theorem:

$$v_{bE} = \sqrt{{v_{br}}^2 + {v_{rE}}^2} = \sqrt{(10.0)^2 + (5.00)^2}\ \mathrm{km/h}$$

$$= 11.2\ \mathrm{km/h}$$

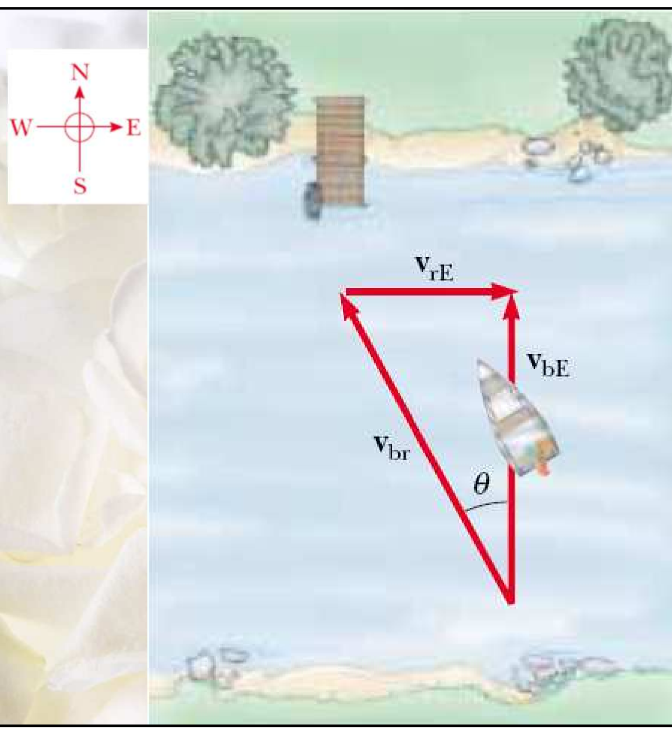
The direction of $\mathbf{v}_{bE}$ is

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{br}}\right) = \tan^{-1}\left(\frac{5.00}{10.0}\right) = 26.6^\circ$$

The boat is moving at a speed of 11.2 km/h in the direction 26.6° east of north relative to Earth. To *finalize* the problem, note that the speed of 11.2 km/h is faster than your boat speed of 10.0 km/h. The current velocity adds to yours to give you a larger speed. Notice in Figure 4.24 that your resultant velocity is at an angle to the direction straight across the river, so you will end up downstream, as we predicted.

Example 4.11 Which Way Should We Head?

If the boat of the preceding example travels with the same speed of 10.0 km/h relative to the river and is to travel due north, as shown in Figure 4.25, what should its heading be?



Solution This example is an extension of the previous one, so we have already *conceptualized* and *categorized* the problem. The *analysis* now involves the new triangle shown in Figure 4.25. As in the previous example, we know $\mathbf{v}_{\mathrm{rE}}$ and the magnitude of the vector $\mathbf{v}_{\mathrm{br}}$, and we want $\mathbf{v}_{\mathrm{bE}}$ to be directed across the river. Note the difference between the triangle in Figure 4.24 and the one in Figure 4.25—the hypotenuse in Figure 4.25 is no longer $\mathbf{v}_{\mathrm{bE}}$. Therefore, when we use the Pythagorean theorem to find $\mathbf{v}_{\mathrm{bE}}$ in this situation, we obtain

$$v_{\mathrm{bE}} = \sqrt{v_{\mathrm{br}}^2 - v_{\mathrm{rE}}^2} = \sqrt{(10.0)^2 - (5.00)^2}\ \mathrm{km/h} = 8.66\ \mathrm{km/h}$$

Now that we know the magnitude of $\mathbf{v}_{\mathrm{bE}}$, we can find the direction in which the boat is heading:

$$\theta = \tan^{-1}\left(\frac{v_{\mathrm{rE}}}{v_{\mathrm{bE}}}\right) = \tan^{-1}\left(\frac{5.00}{8.66}\right) = 30.0^\circ$$

To *finalize* this problem, we learn that the boat must head upstream in order to travel directly northward across the river. For the given situation, the boat must steer a course 30.0° west of north.

What If? **Imagine that the two boats in Examples 4.10 and 4.11 are racing across the river. Which boat arrives at the opposite bank first?**

Answer In Example 4.10, the velocity of 10 km/h is aimed directly across the river. In Example 4.11, the velocity that is directed across the river has a magnitude of only 8.66 km/h. Thus, the boat in Example 4.10 has a larger velocity component directly across the river and will arrive first.