CHAPTER 3
Prelude to Quantum Theory


Max Karl Ernst Ludwig Planck (1858-1947)

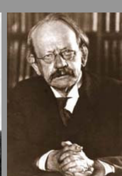
We have no right to assume that any physical laws exist, or if they have existed up until now, or that they will continue to exist in a similar manner in the future.

An important scientific innovation rarely makes its way by gradually winning over and converting its opponents. What does happen is that the opponents gradually die out.

- Max Planck

3.1: Discovery of the X-Ray and the Electron


Wilhelm Röntgen (1845-1923)


J. J. Thomson (1856-1940)

In the 1890s scientists and engineers were familiar with "cathode rays." These rays were generated from one of the metal plates in an evacuated tube with a large electric potential across it.

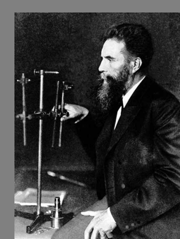
It was surmised that cathode rays had something to do with atoms.

It was known that cathode rays could penetrate matter and were deflected by magnetic and electric fields.

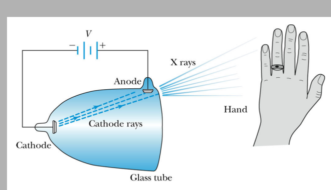
Observation of X Rays

Wilhelm Röntgen studied the effects of cathode rays passing through various materials. He noticed that a phosphorescent screen near the tube glowed during some of these experiments. These new rays were unaffected by magnetic fields and penetrated materials more than cathode rays.

He called them **x rays** and deduced that they were produced by the cathode rays bombarding the glass walls of his vacuum tube.


Wilhelm Röntgen

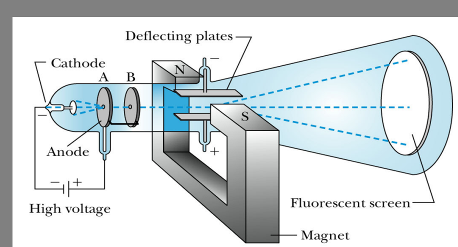
Röntgen's X-Ray Tube



Röntgen constructed an x-ray tube by allowing cathode rays to impact the glass wall of the tube and produced x rays. He used x rays to make a shadowgram the bones of a hand on a phosphorescent screen.

Thomson's Cathode-Ray Experiment

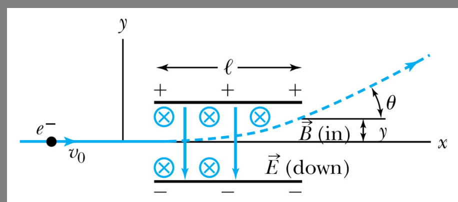
Thomson used an evacuated cathode-ray tube to show that the cathode rays were negatively charged particles (electrons) by deflecting them in electric and magnetic fields.



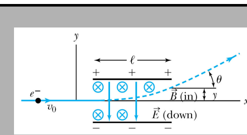
Thomson's Experiment: e/m

Thomson's method of measuring the ratio of the electron's charge to mass was to send electrons through a region containing a magnetic field perpendicular to an electric field.

J. J. Thomson



Calculation of *e*/*m*



An electron moving through the electric field is accelerated by a force: $F_y = ma_y = eE$

Electron angle of deflection: $\tan(\theta) = \frac{v_y}{v_x} = \frac{a_y t}{v_0} = \frac{eE}{m}\frac{\ell}{v_0^2} \longleftarrow t = \frac{\ell}{v_0}$

Then turn on the magnetic field, which deflects the electron against the electric field force. $\vec{F} = e\vec{E} + e\vec{v}_0 \times \vec{B} = 0$

The magnetic field is then adjusted until the net force is zero.

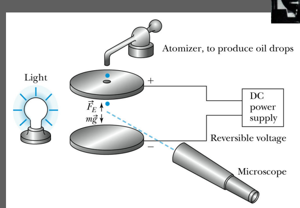
$\vec{E} = -\vec{v}_0 \times \vec{B} \quad \Rightarrow \; v_0 = E/B \quad \Rightarrow \; \tan(\theta) = \frac{eE}{m}\frac{\ell}{(E/B)^2}$

Charge to mass ratio: $\frac{e}{m} = \frac{E\tan(\theta)}{B^2\ell}$

3.2: Determination of Electron Charge

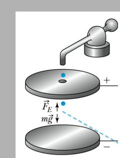
Millikan's oil-drop experiment

Robert Andrews Millikan (1868 – 1953)



Millikan was able to show that electrons had a particular charge.

Calculation of the oil drop charge



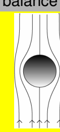
Millikan used an electric field to balance gravity and suspend

$F = 6\pi\eta v$

$6\pi\eta v = mg$

$6\pi\eta v = 4/3\pi r^3 \rho g$

$r = 3\sqrt{\eta v_t / 2g\rho}$



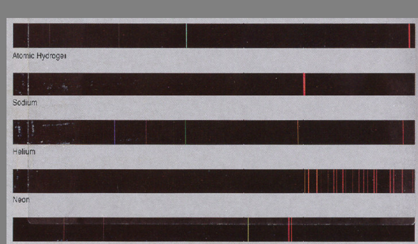
Turning off the electric field, Millikan noted that the drop mass, m_{drop}, could be determined from Stokes' relationship of the terminal velocity, v_t, to the drop density, ρ, and the air viscosity, η :

$$r = 3\sqrt{\eta v_t / 2g\rho} \quad \text{and} \quad m_{drop} = \tfrac{4}{3}\pi r^3 \rho$$

Thousands of experiments showed that there is a basic quantized electron charge: $e = 1.602 \times 10^{-19}$ C

3.3: Line Spectra

Chemical elements were observed to produce unique wavelengths of light when burned or excited in an electrical discharge.

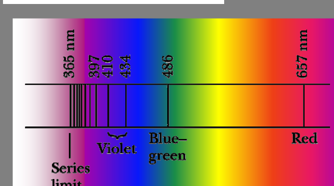


Balmer Series

In 1885, Johann Balmer found an empirical formula for the wavelength of the visible hydrogen line spectra in nm:

$$\lambda = 364.56 \frac{k^2}{k^2 - 4} \text{ nm}$$

(where $k = 3,4,5\ldots$)



Rydberg Equation

As more scientists discovered emission lines at infrared and ultraviolet wavelengths, the Balmer series equation was extended to the Rydberg equation:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{k^2}\right) \qquad R_H = 1.096776 \times 10^7 \text{ m}^{-1}$$

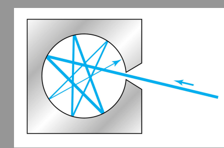
Hydrogen Series of Spectral Lines

Discoverer (year)	Wavelength	n	k
Lyman (1916)	Ultraviolet	1	>1
Balmer (1885)	Visible, ultraviolet	2	>2
Paschen (1908)	Infrared	3	>3
Brackett (1922)	Infrared	4	>4
Pfund (1924)	Infrared	5	>5

3.5: Blackbody Radiation

When matter is heated, it emits radiation.

A blackbody is a cavity with a material that only emits thermal radiation. Incoming radiation is absorbed in the cavity.

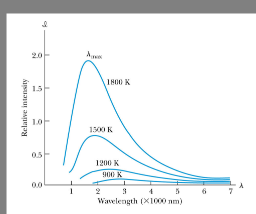


Blackbody radiation is theoretically interesting because the radiation properties of the blackbody **are independent of the particular material**. Physicists can study the properties of intensity versus wavelength at fixed temperatures.

Wien's Displacement Law

The spectral intensity $\mathcal{I}(\lambda, T)$ is the total power radiated per unit area per unit wavelength at a given temperature.

Wien's displacement law: The maximum of the spectrum shifts to smaller wavelengths as the temperature is increased.



$$\lambda_{max} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$$

Stefan-Boltzmann Law

The total power radiated increases with the temperature:

$$P(T) = \int_0^\infty \mathcal{I}(\lambda, T)\,d\lambda = \epsilon\,\sigma T^4$$

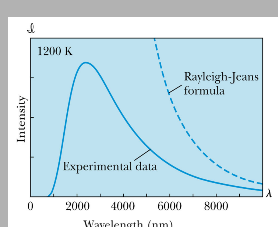
This is known as the **Stefan-Boltzmann law**, with the constant σ experimentally measured to be 5.6705 × 10^{-8} W / (m² · K⁴).

The **emissivity** ϵ (ϵ = 1 for an idealized blackbody) is simply the ratio of the emissive power of an object to that of an ideal blackbody and is always less than 1.

Rayleigh-Jeans Formula

Lord Rayleigh used the classical theories of electromagnetism and thermodynamics to show that the blackbody spectral distribution should be:

$$\mathcal{I}(\lambda, T) = \frac{2\pi c k T}{\lambda^4}$$



It approaches the data at longer wavelengths, but it deviates badly at short wavelengths. This problem for small wavelengths became known as **the ultraviolet catastrophe** and was one of the outstanding exceptions that classical physics could not explain.

Planck's Radiation Law

Planck assumed that the radiation in the cavity was emitted (and absorbed) by some sort of "oscillators." He used Boltzman's statistical methods to arrive at the following formula that fit the blackbody radiation data.

$$\mathcal{I}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Planck's radiation law

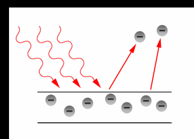
Planck made two modifications to the classical theory:

The oscillators (of electromagnetic origin) can only have certain discrete energies, $E_n = nh\nu$, where n is an integer, ν is the frequency, and h is called Planck's constant: h = 6.6261 × 10^{-34} J·s.

The oscillators can absorb or emit energy in discrete multiples of the fundamental quantum of energy given by:

$$\Delta E = h\nu$$

3.6: Photoelectric Effect



Methods of electron emission:

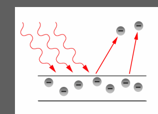
Thermionic emission: Applying heat allows electrons to gain enough energy to escape.

Secondary emission: The electron gains enough energy by transfer from another high-speed particle that strikes the material from outside.

Field emission: A strong external electric field pulls the electron out of the material.

Photoelectric effect: Incident light (electromagnetic radiation) shining on the material transfers energy to the electrons, allowing them to escape. We call the ejected electrons **photoelectrons**.

Photo-electric Effect Experimental Setup



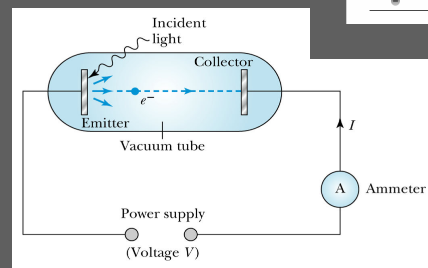
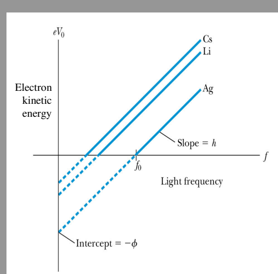


Photo-electric effect observations

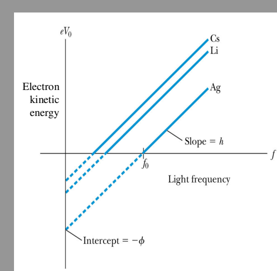


The kinetic energy of the photoelectrons is **independent of the light intensity**.

The kinetic energy of the photoelectrons, for a given emitting material, depends only on the **frequency** of the light.

Classically, the kinetic energy of the photoelectrons should increase with the light intensity and not depend on the frequency.

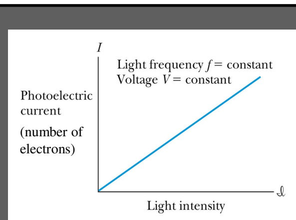
Photo-electric effect observations



There was a **threshold frequency** of the light, below which no photoelectrons were ejected (related to the work function ϕ of the emitter material).

The existence of a threshold frequency is completely inexplicable in classical theory.

Photo-electric effect observations

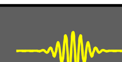


When photoelectrons are produced, their **number** is proportional to the intensity of light.

Also, the photoelectrons are emitted almost **instantly** following illumination of the photocathode, independent of the intensity of the light.

Classical theory predicted that, for extremely low light intensities, a long time would elapse before any one electron could obtain sufficient energy to escape. We observe, however, that the photoelectrons are ejected almost immediately.

Einstein's Theory: Photons



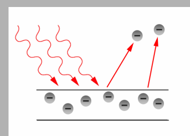
Einstein suggested that the electro-magnetic radiation field is quantized into particles called **photons**. Each photon has the energy quantum:

$$E = h\nu$$

where ν is the frequency of the light and h is Planck's constant. Alternatively,

$$E = \hbar\omega \quad \text{where:} \quad \hbar \equiv h/2\pi$$

Einstein's Theory



Conservation of energy yields:

Energy before (photon) = energy after (electron)

$$h\nu = \phi + \tfrac{1}{2}mv^2$$

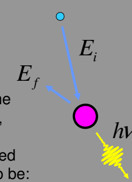
where ϕ is the work function of the metal (potential energy to be overcome before an electron could escape).

In reality, the data were a bit more complex. Because the electron's energy can be reduced by the emitter material, consider v_{max} (not v):

$$h\nu = \phi + \tfrac{1}{2}mv_{max}^2$$

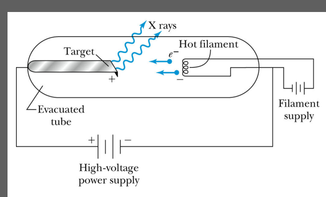
3.7: X-Ray Production: Theory

An energetic electron passing through matter will radiate photons and lose kinetic energy, called bremsstrahlung. Since momentum is conserved, the nucleus absorbs very little energy, and it can be ignored. The final energy of the electron is determined from the conservation of energy to be:



$$E_f = E_i - h\nu$$

X-Ray Production: Experiment



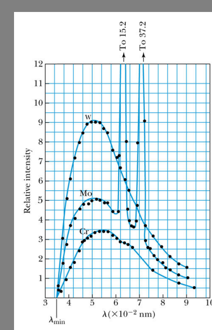
Current passing through a filament produces copious numbers of electrons by thermionic emission. If one focuses these electrons by a cathode structure into a beam and accelerates them by potential differences of thousands of volts until they impinge on a metal anode surface, they produce x rays by bremsstrahlung as they stop in the anode material.

Inverse Photoelectric Effect

Conservation of energy requires that the electron kinetic energy equal the maximum photon energy (neglect the work function because it's small compared to the electron potential energy). This yields the **Duane-Hunt limit**, first found experimentally. The photon wavelength depends only on the accelerating voltage and is the same for all targets.

$$eV_0 = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

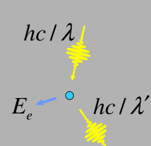
$$\lambda_{min} = \frac{hc}{e}\frac{1}{V_0} = \frac{1.240 \times 10^{-6}\ \mathrm{V \cdot m}}{V_0}$$



3.8: Compton Effect

When a photon enters matter, it can interact with one of the electrons. The laws of conservation of energy and momentum apply, as in any elastic collision between two particles. The momentum of a particle moving at the speed of light is:

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$



The electron energy is: $E_e^2 = (mc^2)^2 + p_e^2c^2$

This yields the change in wavelength of the scattered photon, known as the **Compton effect**:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

3.9: Pair Production and Annihilation

If a photon can create an electron, it must also create a positive charge to balance charge conservation.

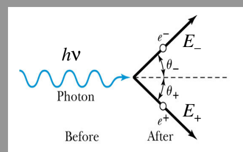
In 1932, C. D. Anderson observed a positively charged electron (e^+) in cosmic radiation. This particle, called a positron, had been predicted to exist several years earlier by P. A. M. Dirac.

Paul Dirac
(1902 - 1984)

A photon's energy can be converted entirely into an electron and a positron in a process called pair production:

$$\gamma \rightarrow e^+ + e^-$$

Pair Production in Empty Space



Conservation of energy for pair production in empty space is:

$$h\nu = E_+ + E_-$$

The total energy for a particle is: $E_\pm^2 = p_\pm^2 c^2 + m^2 c^4$

So: $E_\pm > p_\pm c$

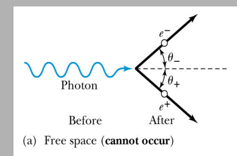
This yields a lower limit on the photon energy: $h\nu > p_- c + p_+ c$

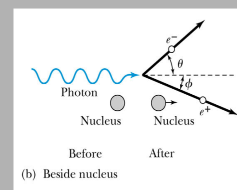
Momentum conservation yields: $h\nu = p_- c\cos(\theta_-) + p_+ c\cos(\theta_+)$

This yields an upper limit on the photon energy: $h\nu < p_- c + p_+ c$

A contradiction! And hence the conversion of energy and momentum for pair production in empty space is impossible!

Pair Production in Matter





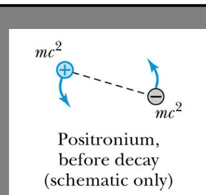
In the presence of matter, the nucleus absorbs some energy and momentum.

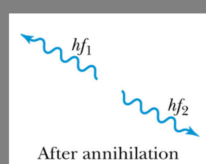
The photon energy required for pair production in the presence of matter is:

$$h\nu = E_+ + E_- + K.E.(nucleus)$$

$$h\nu > 2m_e c^2 = 1.022\ MeV$$

Pair Annihilation

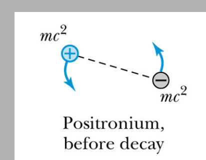


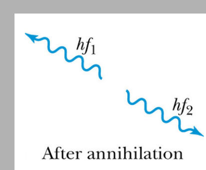


A positron passing through matter will likely **annihilate** with an electron. The electron and positron can form an atom-like configuration first, called **positronium**.

Pair annihilation in empty space produces two photons to conserve momentum. Annihilation near a nucleus can result in a single photon.

Pair Annihilation





Conservation of energy:

$$2m_e c^2 \approx h\nu_1 + h\nu_2$$

Conservation of momentum:

$$\frac{h\nu_1}{c} - \frac{h\nu_2}{c} = 0$$

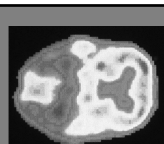
So the two photons will have the same frequency:

$$\nu_1 = \nu_2 = \nu$$

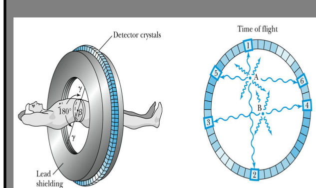
The two photons from positronium annihilation will move in opposite directions with an energy:

$$h\nu = m_e c^2 = 0.511\ \mathrm{MeV}$$

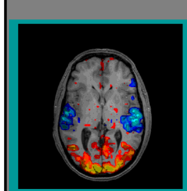
Positron-Emission Tomography

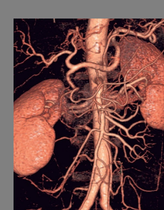


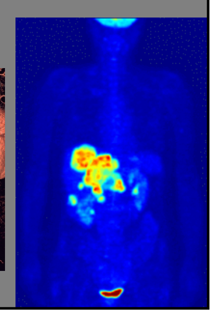
PET scan of a normal brain

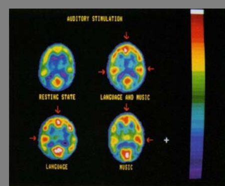


Positron emission tomography (**PET**) is a nuclear medicine imaging technique which produces a three-dimensional image or map of functional processes in the body. The system detects pairs of gamma rays emitted indirectly by a positron-emitting radioisotope, which is introduced into the body on a metabolically active molecule. Images of metabolic activity in space are then reconstructed by computer analysis, often in modern scanners aided by results from a CT X-ray scan performed on the patient at the same time, in the same machine.








AUDITORY STIMULATION
RESTING STATE
LANGUAGE AND MUSIC
LANGUAGE
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