CHAPTER 5

Wave Properties of Matter and Quantum Mechanics I

Louis de Broglie
(1892-1987)

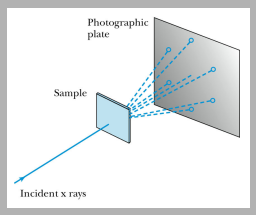
I thus arrived at the overall concept which guided my studies: for both matter and radiations, light in particular, it is necessary to introduce the corpuscle concept and the wave concept at the same time.

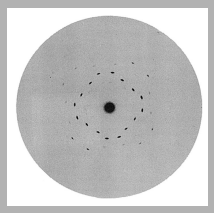
- Louis de Broglie, 1929

5.1: X-Ray Scattering

Max von Laue suggested that if x-rays were a form of electromagnetic radiation, interference effects should be observed.

Crystals act as three-dimensional gratings, scattering the waves and producing observable interference effects.

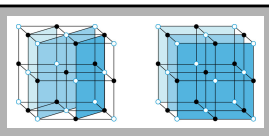




Bragg's Law

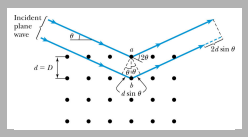
William Lawrence Bragg interpreted the x-ray scattering as the reflection of the incident x-ray beam from a unique set of planes of atoms within the crystal.

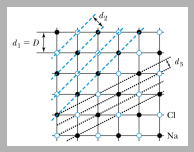
There are two conditions for constructive interference of the scattered x rays:



1) The angle of incidence must equal the angle of reflection of the outgoing wave.
2) The difference in path lengths must be an integral number of wavelengths.

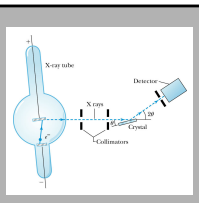
Bragg's Law: $n\lambda = 2d \sin\theta$ (n = integer)



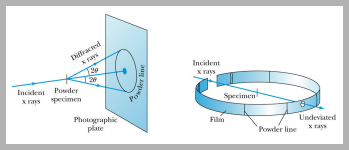


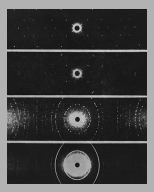
The Bragg Spectrometer

A Bragg spectrometer scatters x rays from crystals. The intensity of the diffracted beam is determined as a function of scattering angle by rotating the crystal and the detector.

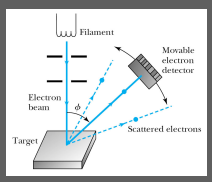


When a beam of x rays passes through a powdered crystal, the dots become a series of rings.



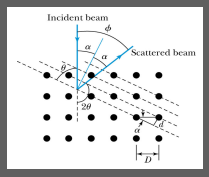


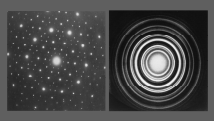
5.3: Electron Scattering



George P. Thomson (1892–1975), son of J. J. Thomson, reported seeing electron diffraction in transmission experiments on celluloid, gold, aluminum, and platinum. A randomly oriented polycrystalline sample of SnO_2 produces rings.

In 1925, Davisson and Germer experimentally observed that **electrons** were diffracted (much like x-rays) in nickel crystals.





5.2: De Broglie Waves

In his thesis in 1923, Prince Louis V. de Broglie suggested that mass particles should have wave properties similar to electromagnetic radiation.

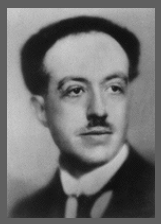
The energy can be written as:

$$h\nu = pc = p\lambda\nu$$

Thus the wavelength of a matter wave is called **the de Broglie wavelength**:

$$\lambda = \frac{h}{p}$$

If a light-wave could also act like a particle, why shouldn't matter-particles also act like waves?



Louis V. de Broglie
(1892-1987)

Bohr's Quantization Condition revisited

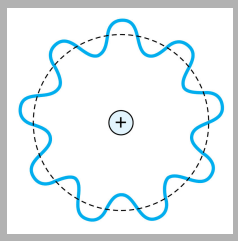
One of Bohr's assumptions in his hydrogen atom model was that the angular momentum of the electron in a stationary state is $n\hbar$.

This turns out to be equivalent to saying that the electron's orbit consists of an integral number of electron de Broglie wavelengths:

$$2\pi r = n\lambda = n\frac{h}{p}$$

electron de Broglie wavelength

Circumference



Multiplying by $p/2\pi$, we find the angular momentum:

$$L = rp = \frac{nh}{2\pi} = n\hbar$$

5.4: Wave Motion

De Broglie matter waves should be described in the same manner as light waves. The matter wave should be a solution to a wave equation like the one for electromagnetic waves:

$$\frac{\partial^2\Psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2} = 0$$

It will actually be different, but, in some cases, the solutions are the same.

with a solution:

$$\Psi(x,t) = A\exp[i(kx - \omega t - \theta)]$$

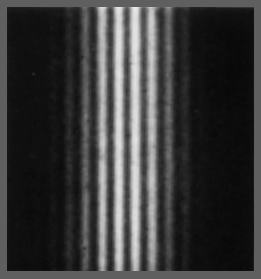
Define the wave number k and the angular frequency ω as usual:

$$k = \frac{2\pi}{\lambda} \text{ and } \omega = \frac{2\pi}{T}$$

Electron Double-Slit Experiment

C. Jönsson of Tübingen, Germany, succeeded in 1961 in showing double-slit interference effects for **electrons** by constructing very narrow slits and using relatively large distances between the slits and the observation screen.

This experiment demonstrated that precisely the same behavior occurs for both light (waves) and electrons (particles).



Wave-particle-duality solution

It's somewhat disturbing that everything is both a particle and a wave.

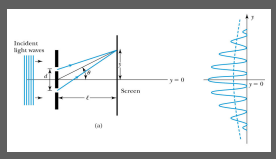
The wave-particle duality is a little less disturbing if we think in terms of:

Bohr's Principle of Complementarity: It's not possible to describe physical observables **simultaneously** in terms of both particles and waves.

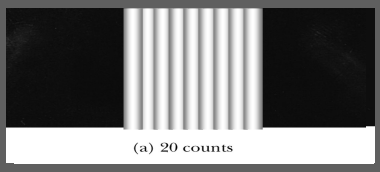
When we're making a measurement, use the particle description, but when we're not, use the wave description.

When we're looking, fundamental quantities are particles; when we're not, they're waves.

5.5: Waves or Particles?



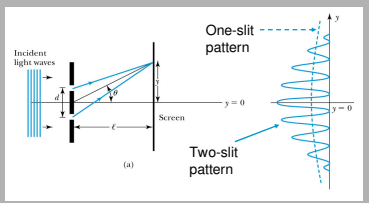
Dimming the light in Young's two-slit experiment results in single photons at the screen. Since photons are particles, each can only go through one slit, so, at such low intensities, their distribution should become the single-slit pattern.



Each photon actually goes through **both** slits!

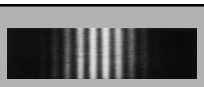
Can you tell which slit the photon went through in Young's double-slit exp't?

When you block one slit, the one-slit pattern returns.



At low intensities, Young's two-slit experiment shows that light propagates as a wave and is detected as a particle.

Which slit does the electron go through?



Shine light on the double slit and observe with a microscope. After the electron passes through one of the slits, light bounces off it; observing the reflected light, we determine which slit the electron went through.

The photon momentum is: $p_{ph} = \frac{h}{\lambda_{ph}} > \frac{h}{d}$ ← Need $\lambda_{ph} < d$ to distinguish the slits.

The electron momentum is: $p_{el} = \frac{h}{\lambda_{el}} \sim \frac{h}{d}$ ← Diffraction is significant only when the aperture is ~ the wavelength of the wave.

The momentum of the photons used to determine which slit the electron went through is enough to strongly modify the momentum of the electron itself—changing the direction of the electron! The attempt to identify which slit the electron passes through will in itself change the diffraction pattern! Electrons also propagate as waves and are detected as particles.

5.6: Uncertainty Principle: Energy Uncertainty

The energy uncertainty of a wave packet is:

$$\Delta E = h\,\Delta\nu = h\frac{\Delta\omega}{2\pi} = \hbar\,\Delta\omega$$

Combined with the angular frequency relation we derived earlier:

$$\Delta\omega\,\Delta t = \frac{\Delta E}{\hbar}\Delta t = \frac{1}{2}$$

Energy-Time Uncertainty Principle: .

$$\boxed{\Delta E\,\Delta t \geq \frac{\hbar}{2}}$$

Momentum Uncertainty Principle

The same mathematics relates x and k: $\boxed{\Delta k\,\Delta x \geq ½}$

So it's also impossible to measure simultaneously the precise values of k and x for a wave.

Now the momentum can be written in terms of k:

$$p = \frac{h}{\lambda} = \frac{h}{2\pi/k} = (h/2\pi)k \quad \Rightarrow \quad \boxed{p = \hbar k}$$

So the uncertainty in momentum is: $\Delta p = \hbar\,\Delta k$

But multiplying $\Delta k\,\Delta x \geq ½$ by $\hbar$: $\hbar\,\Delta k\,\Delta x \geq \frac{\hbar}{2}$

And we have **Heisenberg's Uncertainty Principle**: $\boxed{\Delta p_x\,\Delta x \geq \frac{\hbar}{2}}$

How to think about Uncertainty

The act of making one measurement perturbs the other.

Precisely measuring the time disturbs the energy.

Precisely measuring the position disturbs the momentum.

The Heisenbergmobile. The problem was that when you looked at the speedometer you got lost.

Kinetic Energy Minimum

Since we're always uncertain as to the exact position, $\Delta x = \ell$, of a particle, for example, an electron somewhere inside an atom, the particle can't have zero kinetic energy:

$$\Delta p \geq \frac{\hbar}{\Delta x} = \frac{\hbar}{\ell}$$

The average of a positive quantity must always exceed its uncertainty:

$$p_{ave} \geq \Delta p \geq \frac{\hbar}{\Delta x} = \frac{\hbar}{\ell}$$

so:

$$K_{ave} = \frac{p_{ave}^2}{2m} \geq \frac{(\Delta p)^2}{2m} \geq \frac{\hbar^2}{2m\ell}$$

5.7: Probability, Wave Functions, and the Copenhagen Interpretation

Okay, if particles are also waves, what's waving? Probability

The wave function determines the likelihood (or probability) of finding a particle at a particular position in space at a given time:

$\boxed{P(x) = |\Psi(x)|^2}$ The probability of the particle being between x_1 and x_2 is given by: $\boxed{\int_{x_1}^{x_2} |\Psi(x)|^2\,dx}$

$\boxed{\int_{-\infty}^{\infty} |\Psi(x)|^2\,dx = 1}$ The total probability of finding the particle is 1. Forcing this condition on the wave function is called normalization.

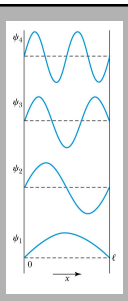
5.8: Particle in a Box

A particle (wave) of mass m is in a one-dimensional box of width l.

The box puts boundary conditions on the wave. The wave function must be zero at the walls of the box and on the outside.

In order for the probability to vanish at the walls, we must have an integral number of half wavelengths in the box:

$$\frac{n\lambda}{2} = \ell \quad \text{or} \quad \lambda_n = \frac{2\ell}{n} \quad (n = 1, 2, 3, \ldots)$$



The energy:

$$E = K.E. = \tfrac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

The possible wavelengths are quantized and hence so are the energies:

$$E_n = \frac{h^2}{2m}\left(\frac{n}{2\ell}\right)^2 = n^2\frac{h^2}{8m\ell^2} \quad (n = 1, 2, 3, \ldots)$$

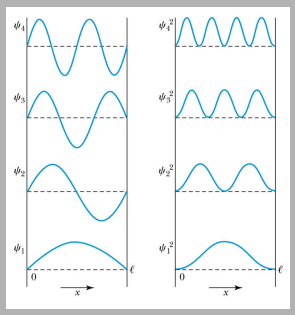
Probability of the particle vs. position

Note that $E_0 = 0$ is **not** a possible energy level.

The concept of energy levels, as first discussed in the Bohr model, has surfaced in a natural way by using waves.

The probability of observing the particle between x and $x + dx$ in each state is

$$P_n\,dx \propto |\Psi_n(x)|^2\,dx$$



The Copenhagen Interpretation

Bohr's interpretation of the wave function consisted of three principles:

- Heisenberg's uncertainty principle
- Bohr's complementarity principle
- Born's statistical interpretation, based on probabilities determined by the wave function

Together these three concepts form a logical interpretation of the physical meaning of quantum theory. In the Copenhagen interpretation, **physics describes only the results of measurements**.