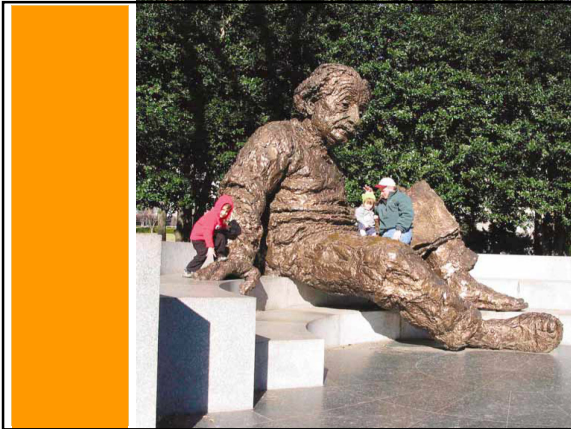




Relativity

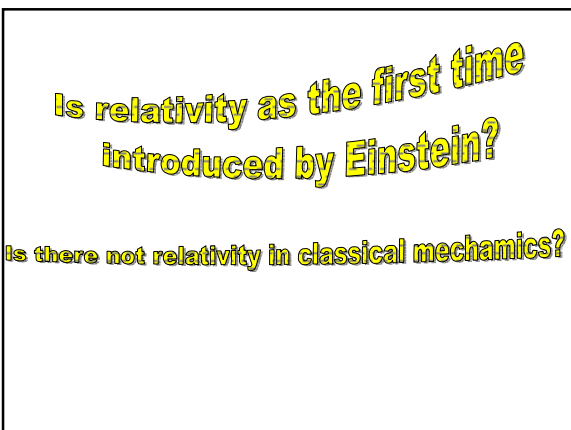
- We can only use physics when we have a frame of reference to *describe* what is happening

Relativity

- We can only use physics when we have a frame of reference to *describe* what is happening
- However, frames of reference are arbitrary, so we don't want any explanations (theories) to depend on which frame we are using
 - The laws of Physics should be the same in all *inertial* reference frames

Relativity

- We can only use physics when we have a frame of reference to *describe* what is happening
- However, frames of reference are arbitrary, so we don't want any explanations (theories) to depend on which frame we are using
 - The laws of Physics should be the same in all *inertial* reference frames
- Relativity is the study of how one reference frame *relates* to another

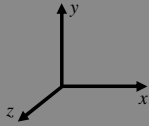


The Principle of Galilean Relativity

The laws of mechanics must be the same in all inertial frames of reference.

Newtonian (Classical) Relativity

Newton's laws of motion must be implemented with respect to (relative to) some reference frame.

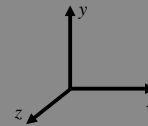


- A reference frame is called an **inertial frame** if Newton's laws are valid in that frame.
- Such a frame is established when a body, not subjected to net external forces, moves in rectilinear motion **at constant velocity**.

Newtonian Principle of Relativity

• If Newton's laws are valid in one reference frame, then they are also valid in another reference frame moving at a uniform velocity relative to the first system.

• This is referred to as the **Newtonian principle of relativity** or **Galilean invariance**.

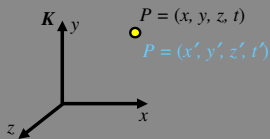


If the axes are also parallel, these frames are said to be **Inertial Coordinate Systems**

The Galilean Transformation

• For a point P:

- In one frame K: $P = (x, y, z, t)$
- In another frame K': $P = (x', y', z', t')$



Conditions of the Galilean Transformation

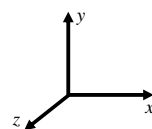
- 1. Parallel axes
- 2. K' has a constant relative velocity (here in the x-direction) with respect to K.
- 3. Time (t) for all observers is a **Fundamental invariant**, i.e., it's the same for all inertial observers.

$$\begin{aligned}x' &= x - vt \\y' &= y \\z' &= z \\t' &= t\end{aligned}$$

The Inverse Relations

- Step 1. Replace $-v$ with $+v$.
- Step 2. Replace "primed" quantities with "unprimed" and "unprimed" with "primed."

$$\begin{aligned}x &= x' + vt' \\y &= y' \\z &= z' \\t &= t'\end{aligned}$$



$$\begin{aligned}x &= x' + vt' \\y &= y' \\z &= z' \\t &= t'\end{aligned}$$

$$\begin{aligned}x_A &= x'_A + vt'_A \\x_B &= x'_B + vt'_B\end{aligned}$$

$$x_B - x_A = x'_B - x'_A + v(t'_B - t'_A)$$

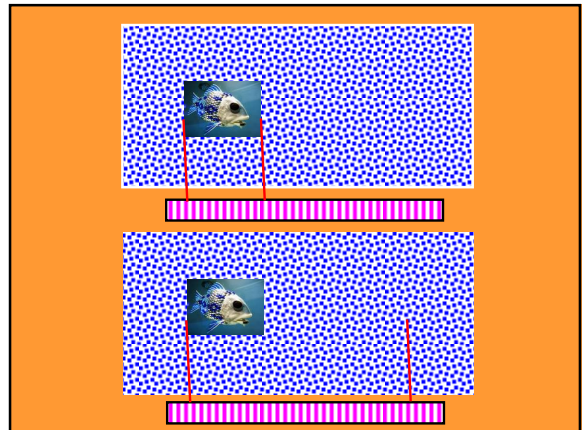
$$x_B - x_A = x'_B - x'_A$$

- Notice that each observers perform two measurements:
1: (x_A, t_A) 2: (x_B, t_B) and 1: (x'_A, t'_A) 2: (x'_B, t'_B)

These measurements are performed at the same time:

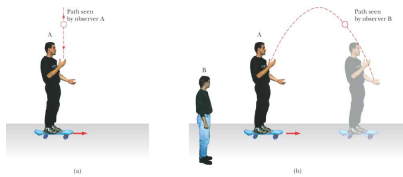
$$t_A = t_B \text{ and } t'_A = t'_B$$

The later assumption (simultaneous measurements) plays an important role for the definition of the length of the movable rode.



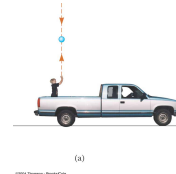
Relativity

- Galilean Relativity, that is...
- What happens when two observers moving at constant relative velocity make observations?
 - Observer A sees only vertical motion
 - Observer B sees a parabolic trajectory (projectile motion)



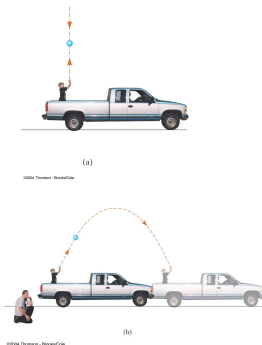
Galilean Relativity

- Start with an example
 - The child in the truck throws the ball *straight* up, and then catches it
 - To the child there is no change in x



Galilean Relativity

- Start with an example
 - The child in the truck throws the ball *straight* up, and then catches it
 - To the child there is no change in x
 - The truck is really moving however, and the man on the ground sees the child throw the ball up and forward in just such a way as to catch it when it comes back down

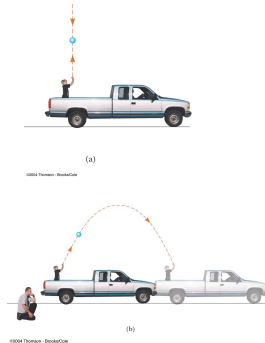


Quick Quiz 39.1 Which observer in Figure 39.1 sees the ball's *correct* path?
(a) the observer in the truck (b) the observer on the ground (c) both observers.

39.1 (c). While the observers' measurements differ, both are correct.

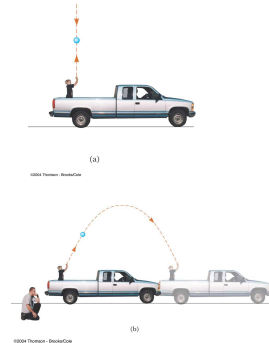
Galilean Relativity

- Who's right?



Galilean Relativity

- Who's right?
 - They both are, of course
- Newton's Laws describe the acceleration of the ball, and both observers agree about the acceleration!

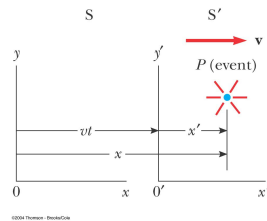


Galilean Relativity

- Let's call the ground frame S and the truck frame S'

How do we relate an event in described in S to the same event described in S' ?

$$\begin{aligned}x' &= x - vt \\y' &= y \\z' &= z \\t' &= t\end{aligned}$$

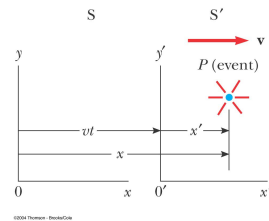


Galilean Relativity

- Let's call the ground frame S and the truck frame S'

What about a series of events (like the velocity and acceleration of the ball)

$$\begin{aligned}\frac{dx'}{dt'} &= \frac{dx}{dt} - v \Rightarrow u' = u - v \\ \frac{dy'}{dt'} &= \frac{dy}{dt} \\ \frac{dz'}{dt'} &= \frac{dz}{dt}\end{aligned}$$



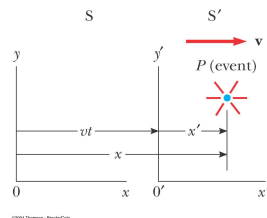
Galilean Relativity

- Let's call the ground frame S and the truck frame S'

What about a series of events (like the velocity and acceleration of the ball)

$$\begin{aligned}\frac{d^2x'}{dt'^2} &= \frac{du'}{dt} = \frac{du}{dt} = \frac{dv}{dt} = \frac{du}{dt} \\ \frac{d^2x'}{dt'^2} &= \frac{d^2x}{dt^2}\end{aligned}$$

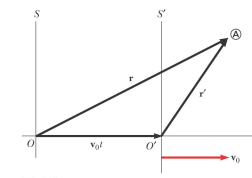
So Newton's Laws are valid in all inertial ($dv/dt=0$) frames!



Galilean Relativity

- As usual, one has to convert from one reference frame to another
 - Origin of S' frame moves with velocity v_0 with respect to frame S

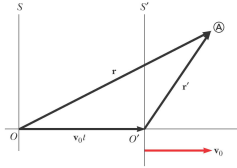
$$\begin{aligned}\mathbf{r}' &= \mathbf{r} - \mathbf{v}_0 t \\ \mathbf{v}' &= \frac{d\mathbf{r}'}{dt} = \frac{d\mathbf{r}}{dt} - \mathbf{v}_0 \\ &= \mathbf{v} - \mathbf{v}_0 \\ \mathbf{a}' &= \frac{d\mathbf{v}'}{dt} = \frac{d\mathbf{v}}{dt} - \frac{d\mathbf{v}_0}{dt} = \frac{d\mathbf{v}}{dt} - 0 \\ &= \mathbf{a}\end{aligned}$$



Galilean Relativity

- Note that acceleration does *not* change!
 - Since (as we'll see) force is proportional to acceleration, (Newtonian) physics is the same in any two frames moving at constant relative velocity!

$$\begin{aligned}\mathbf{r}' &= \mathbf{r} - \mathbf{v}_0 t \\ \mathbf{v}' &= \frac{d\mathbf{r}'}{dt} = \frac{d\mathbf{r}}{dt} - \mathbf{v}_0 \\ &= \mathbf{v} - \mathbf{v}_0 \\ \mathbf{a}' &= \frac{d\mathbf{v}'}{dt} = \frac{d\mathbf{v}}{dt} - \frac{d\mathbf{v}_0}{dt} = \frac{d\mathbf{v}}{dt} - 0 \\ &= \mathbf{a}\end{aligned}$$



Quick Quiz 39.2 A baseball pitcher with a 90-mi/h fastball throws a ball while standing on a railroad flatcar moving at 110 mi/h. The ball is thrown in the same direction as that of the velocity of the train. Applying the Galilean velocity transformation equation, the speed of the ball relative to the Earth is (a) 90 mi/h (b) 110 mi/h (c) 20 mi/h (d) 200 mi/h (e) impossible to determine.

39.2 (d). The Galilean velocity transformation gives us $u_x = u'_x + v = 110 \text{ mi/h} + 90 \text{ mi/h} = 200 \text{ mi/h}$.

Maxwell's Equations & Absolute Reference Systems

- In Maxwell's theory, the speed of light, in terms of the permeability and permittivity of free space, was given by:

$$v = c = 1 / \sqrt{\mu_0 \epsilon_0}$$

Thus the velocity of light is a constant.

Aether was proposed as an absolute reference system in which the speed of light was this constant and from which other measurements could be made.

The Michelson-Morley experiment was an attempt to show the existence of aether.

What about E & M?

- Recall the wave equation for EM waves

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2}$$

This comes from electricity
This comes from magnetism This is the velocity of the wave

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- Does the velocity of the wave transform via Galilean Relativity?
 - It seems that the velocity of light should be transformed depending on what frame we are in
 - But nothing in the equations tell us which is the frame in which $c = 3.00 \times 10^8 \text{ m/s}$.

What about E & M?

- Recall the wave equation for EM waves

$$\frac{\partial^2 E_x}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2}$$

This comes from electricity
This comes from magnetism This is the velocity of the wave

- Does the velocity of the wave transform via Galilean Relativity?
 - It seems that the velocity of light should be transformed depending on what frame we are in
 - But nothing in the equations tell us which is the frame in which $c=3.00 \times 10^8$ m/s.
 - Perhaps light moves through a medium, ether, how can we tell?

The Maxwell's equations

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \rho / \epsilon & \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{B} &= 0 & \vec{\nabla} \times \vec{B} &= \mu \epsilon \frac{\partial \vec{E}}{\partial t} \end{aligned}$$

where $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, ρ is the charge density, ϵ is the permittivity, and μ is the permeability of the medium.

Derivation of the Wave Equation from Maxwell's Equations

Take $\vec{\nabla} \times$ of: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = \vec{\nabla} \times \left[-\frac{\partial \vec{B}}{\partial t} \right]$$

Change the order of differentiation on the RHS:

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\frac{\partial}{\partial t} [\vec{\nabla} \times \vec{B}]$$

Derivation of the Wave Equation from Maxwell's Equations (continued)

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\frac{\partial}{\partial t} [\vec{\nabla} \times \vec{B}]$$

$$\vec{\nabla} \times \vec{B} = \mu \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\frac{\partial}{\partial t} \left[\mu \epsilon \frac{\partial \vec{E}}{\partial t} \right]$$

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{assuming that } \mu \text{ and } \epsilon \text{ are constant in time.}$$

Derivation of the Wave Equation from Maxwell's Equations (cont'd)

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{f}] = \vec{\nabla}(\vec{\nabla} \cdot \vec{f}) - \nabla^2 \vec{f}$$

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

•Galilean transformation is not valid for those principles of physics that contains velocity.

•If we accept that both the Galilean transformation and Maxwell equation are valid, then it is spontaneously concluded that there is a unique preferable frame.

•The following possibilities is occurred by the fact that the Galilean relativity is satisfied by the Newton laws but it is not satisfied by the Maxwell equation:

- There is relativity principle for the Mechanics, but for the electrodynamics. This requires that there is a unique and preferable fram. If this is so we can apply the Galilean transformation and find the preferable fram.
- There is a relativity principle that are valid for both Mechanics and Electrodynamics, but the Maxwell equation is incorrect and should be represented. If this is so we should be able to observe some deviation from Maxwell equation.
- There is a relativity principle that are valid for both Mechanics and Electrodynamics, but the laws of Newtonian mechanics are not correct. If this is so, then we should be able to observe deviation from classical mechanics, and represent the classical mechanics from scratch. In this case the Galilean transformation can not be correct and we should be able to represent it.