آیا مکانیک کلاسیک تحت وارونی زمان ناوردا باقی می ماند؟

الکترومغناطیس چطور؟

مکانیک کوانتمی چطور؟

در چه صورت مکانیک کوانتمی تحت وارونی زمان ناوردا باقی می ماند؟

آیا عملگر وارونی زمان یک عملگر یونیتاری است؟

آیا ضرورتی دارد که طول بردار تحت وارونی زمان ناوردا باقی بماند؟

عملگر پاد یونیتاری چیست؟

THE TIME-REVERSAL DISCRETE SYMMETRY

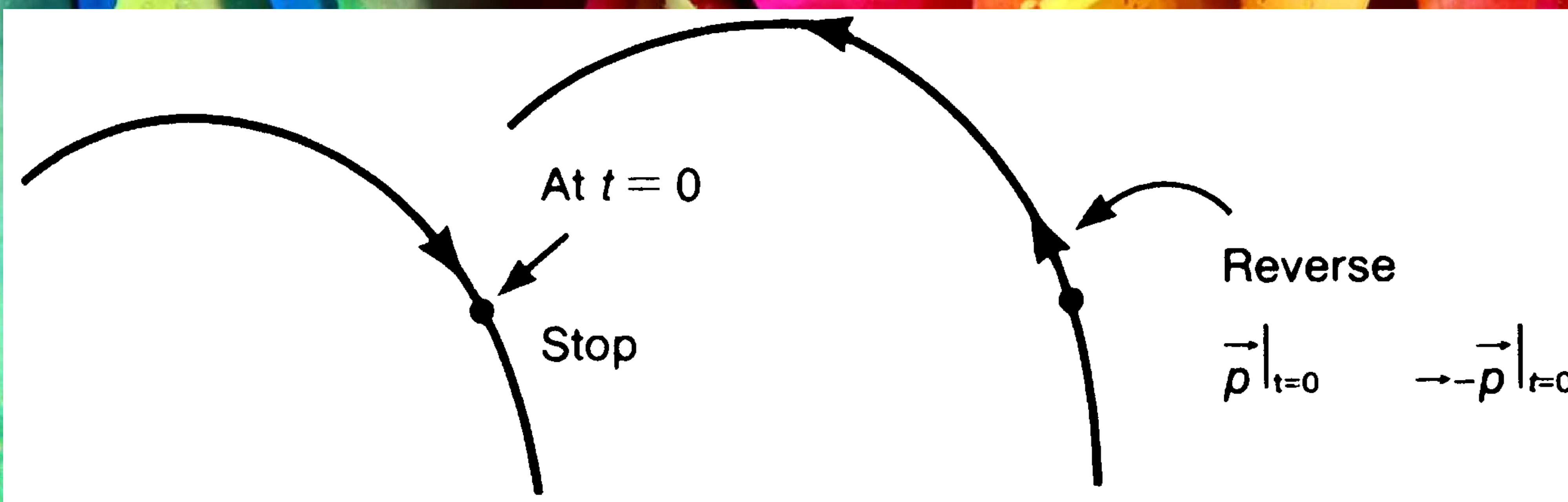
In this section we study another discrete symmetry operator, called **time reversal**.

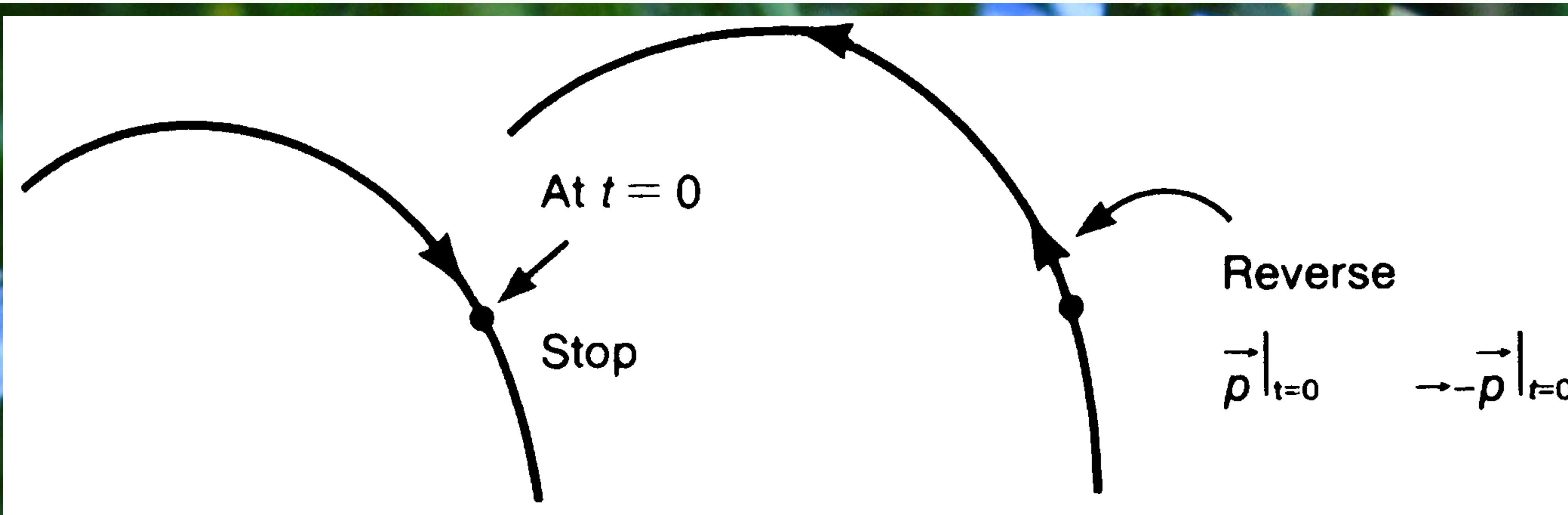
This is a difficult topic for the novice, partly because the term time reversal is a misnomer; it reminds us of science fiction.

Actually what we do in this section can be more appropriately characterized by the term reversal of motion.

For orientation purposes let us look at classical mechanics.

Suppose there is a trajectory of a particle subject to a certain force field; see Figure 4.9.





At t = 0, let the particle stop and reverse its motion: $p|_{t=0} \rightarrow -p|_{t=0}$.

The particle traverses backward along the same trajectory.

If you run the motion picture of trajectory (a) backward as in (b), you may have a hard time telling whether this is the correct sequence.

More formally, if x(t) is a solution to

$$m\ddot{\mathbf{x}} = -\nabla V(\mathbf{x})$$

then x(-t) is also a possible solution in the same force field derivable from V.

It is, of course, important to note that we do not have a dissipative force here.

A block sliding on a table decelerates (due to friction) and eventually stops.

But have you ever seen a block on a table spontaneously start to move and accelerate?

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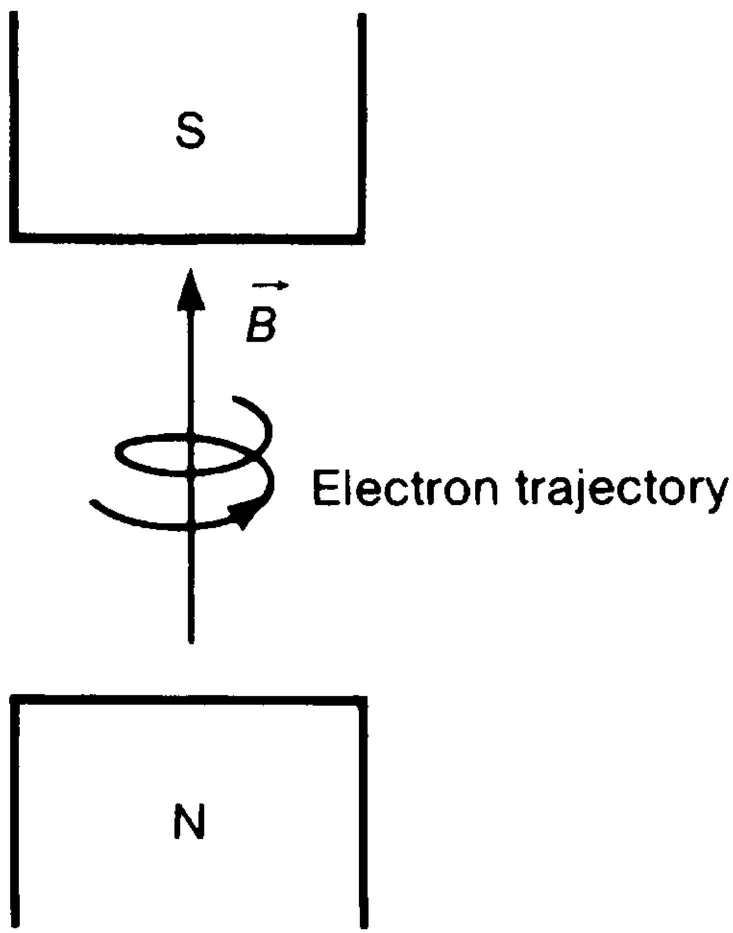
With a magnetic field you may be able to tell the difference.

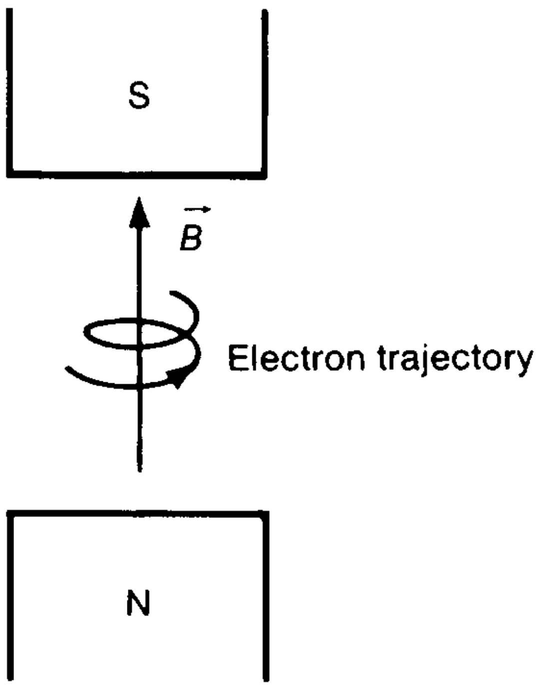
Imagine that you are taking the motion picture of a spiraling electron trajectory in a magnetic field.



You may be able to tell whether the motion picture is run forward or backward by comparing the sense of rotation with the magnetic pole labeling N and S.

However, from a microscopic point of view, B is produced by moving charges via an electric current; if you could reverse the current that causes B, then the situation would be quite symmetrical.

In terms of the picture shown in Figure 4.10, you may have figured out that N and S are mislabeled!



Another more formal way of saying all this is that the Maxwell equations, for example,

$$\nabla\cdot\mathbf{E} = 4\pi\rho, \quad \nabla\times\mathbf{B} - \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} = \frac{4\pi\mathbf{j}}{c}, \quad \nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},$$

and the Lorentz force equation $\boldsymbol{F} = e[\boldsymbol{E} + \frac{1}{c}(\boldsymbol{v} \times \boldsymbol{B})$ are invariant under t →-t provided we also let

$$\mathbf{E}\to\mathbf{E}, \qquad \mathbf{B}\to-\mathbf{B}, \qquad \rho\to\rho, \qquad \mathbf{j}\to-\mathbf{j}, \qquad \mathbf{v}\to-\mathbf{v}.$$

Let us now look at wave mechanics, where the basic equation of the Schrodinger wave equation is

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi$$

Suppose $\psi(x,t)$ is a solution.

We can easily verify that $\psi(x,-t)$ is not a solution, because of the appearance of the first-order time derivative.

However, $\psi^*(x,-t)$ is a solution, as you may verify by complex conjugation of $i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi$.

Thus we conjecture that time reversal must have something to do with complex conjugation.

If at t = 0 the wave function is given by $\psi = \langle x|\alpha\rangle$ then the wave function for the corresponding time-reversed state is given by $\langle x|\alpha\rangle^*$.

We will later show that this is indeed the case for the wave function of a spinless system.

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Digression on Symmetry Operations

Before we begin a systematic treatment of the time-reversal operator, some general remarks on symmetry operations are in order.

Consider a symmetry operation

$$|\alpha\rangle \rightarrow |\tilde{\alpha}\rangle, \qquad |\beta\rangle \rightarrow |\tilde{\beta}\rangle$$

One may argue that it is natural to require the inner product $\langle\beta|\alpha\rangle$ to be preserved, that is,

$$\langle\tilde{\beta}|\tilde{\alpha}\rangle = \langle\beta|\alpha\rangle$$

Indeed, for symmetry operations such as rotations, translations, and even parity, this is indeed the case.

If $|\alpha\rangle$ is rotated and $|\beta\rangle$ is also rotated in the same manner, $\langle\beta|\alpha\rangle$ is unchanged.

Formally this arises from the fact that, for the symmetry operations considered in the previous sections, the corresponding symmetry operator is unitary, so

$$\langle\beta|\alpha\rangle \to \langle\beta|U^{\dagger}U|\alpha\rangle = \langle\beta|\alpha\rangle$$

However, in discussing time reversal, we see that requirement $\langle\tilde{\beta}|\tilde{\alpha}\rangle = \langle\beta|\alpha\rangle$ turns out to be too restrictive.

Instead, we merely impose the weaker requirement that

$$|\langle\tilde{\beta}|\tilde{\alpha}\rangle| = |\langle\beta|\alpha\rangle|$$

Requirement $\langle\tilde{\beta}|\tilde{\alpha}\rangle = \langle\beta|\alpha\rangle$ obviously satisfies $|\langle\tilde{\beta}|\tilde{\alpha}\rangle| = |\langle\beta|\alpha\rangle|$.

But this is not the only way; $\langle\tilde{\beta}|\tilde{\alpha}\rangle = \langle\beta|\alpha\rangle^* = \langle\alpha|\beta\rangle$ works equally well.

If $\langle\tilde{\beta}|\tilde{\alpha}\rangle = \langle\beta|\alpha\rangle^*$ then we have

$$\langle\tilde{\beta}|\tilde{\alpha}\rangle^* = \langle\beta|\alpha\rangle$$

Thus we can write

$$\langle\tilde{\beta}|\tilde{\alpha}\rangle\langle\tilde{\beta}|\tilde{\alpha}\rangle^* = \langle\beta|\alpha\rangle\langle\beta|\alpha\rangle^*$$

But this requires that

$$|\langle\tilde{\beta}|\tilde{\alpha}\rangle| = |\langle\beta|\alpha\rangle|$$

So it is not necessary to have $\langle\tilde{\beta}|\tilde{\alpha}\rangle = \langle\beta|\alpha\rangle$

It is enough to have $\langle\tilde{\beta}|\tilde{\alpha}\rangle = \langle\beta|\alpha\rangle^*$

We pursue the latter possibility in this section because from our earlier discussion based on the Schrodinger equation we inferred that time reversal has something to do with complex conjugation.

Definition.

The transformation $|\alpha\rangle \to |\tilde{\alpha}\rangle = \theta|\alpha\rangle$, $|\beta\rangle \to |\tilde{\beta}\rangle = \theta|\beta\rangle$ is said to be ***antiunitary*** if

$$\langle \tilde{\beta}|\tilde{\alpha}\rangle = \langle \beta|\alpha\rangle^*,$$

$$\theta(c_1|\alpha\rangle + c_2|\beta\rangle) = c_1^*\theta|\alpha\rangle + c_2^*\theta|\beta\rangle$$

In such a case the operator θ is an antiunitary operator.

Relation $\theta(c_1|\alpha\rangle + c_2|\beta\rangle) = c_1^*\theta|\alpha\rangle + c_2^*\theta|\beta\rangle$ alone defines an antilinear operator.

We now claim that an antiunitary operator can be written as $\theta = UK$, where U is a unitary operator and K is the complex-conjugate operator that forms the complex conjugate of any coefficient that multiplies a ket (and stands on the right of K).

Before checking $\langle\tilde{\beta}|\tilde{\alpha}\rangle = \langle\beta|\alpha\rangle^*$ $\theta(c_1|\alpha\rangle + c_2|\beta\rangle) = c_1^*\theta|\alpha\rangle + c_2^*\theta|\beta\rangle$ let us examine the property of the K operator.

Suppose we have a ket multiplied by a complex number c.

We have then $Kc|\alpha\rangle = c^* K|\beta\rangle$.

One may further ask, What happens if |α> is expanded in terms of base kets $\{|a'\rangle\}$?

$$|\alpha\rangle = \sum_{a'} |a'\rangle\langle a'|\alpha\rangle \xrightarrow{K} |\tilde{\alpha}\rangle = \sum_{a'} \langle a'|\alpha\rangle^* K |a'\rangle$$
$$= \sum_{a'} \langle a'|\alpha\rangle^* |a'\rangle .$$

Notice that K acting on the base ket does not change the base ket.

The explicit representation of $|a'\rangle$ is

$$|a'\rangle = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

and there is nothing to be changed by K.

The reader may wonder, for instance, whether the S_y eigenkets for a spin $\frac{1}{2}$ system change under K.

The answer is that if the Sz eigenkets are used as base kets, we must change the Sy eigenkets because the Sy eigenkets $\left|S_y;\pm\right\rangle = \frac{1}{\sqrt{2}}\left|+\right\rangle \pm \frac{i}{\sqrt{2}}\left|-\right\rangle$ undergo under *K*

$$K\left(\frac{1}{\sqrt{2}}|+\rangle \pm \frac{i}{\sqrt{2}}|-\rangle\right) \rightarrow \frac{1}{\sqrt{2}}|+\rangle \mp \frac{i}{\sqrt{2}}|-\rangle$$

On the other hand, if the Sy eigenkets themselves are used as the base kets, we do not change the Sy eigenkets under the action of K.

As a result, the form of U in $\theta = UK$ also depends on the particular representation (that is, the choice of base kets) used.

Gottfried puts it aptly: "If the basis is changed, the work of U and K has to be reapportioned."

Returning to $\theta = UK$ and $\langle\tilde{\beta}|\tilde{\alpha}\rangle = \langle\beta|\alpha\rangle^*$, $\theta(c_1|\alpha\rangle + c_2|\beta\rangle) = c_1^*\theta|\alpha\rangle + c_2^*\theta|\beta\rangle$, let us first check property $\theta(c_1|\alpha\rangle + c_2|\beta\rangle) = c_1^*\theta|\alpha\rangle + c_2^*\theta|\beta\rangle$.

We have

$$|\alpha\rangle \xrightarrow{\theta} |\tilde{\alpha}\rangle = \sum_{a'} \langle a'|\alpha\rangle^* UK|a'\rangle$$
$$= \sum_{a'} \langle a'|\alpha\rangle^* U|a'\rangle$$
$$= \sum_{a'} \langle \alpha|a'\rangle U|a'\rangle.$$

As for $|\beta\rangle$, we have

$$|\tilde{\beta}\rangle = \sum_{a'} \langle a'|\beta\rangle^* U|a'\rangle \overset{\text{DC}}{\leftrightarrow} \langle\tilde{\beta}| = \sum_{a'} \langle a'|\beta\rangle\langle a'|U^\dagger$$
$$\langle\tilde{\beta}|\tilde{\alpha}\rangle = \sum_{a''}\sum_{a'} \langle a''|\beta\rangle\langle a''|U^\dagger U|a'\rangle\langle\alpha|a'\rangle$$
$$= \sum_{a'} \langle\alpha|a'\rangle\langle a'|\beta\rangle = \langle\alpha|\beta\rangle$$
$$= \langle\beta|\alpha\rangle^*,$$

so this checks.

In order for $|\langle\tilde{\beta}|\tilde{\alpha}\rangle| = |\langle\beta|\alpha\rangle|$ to be satisfied, it is of physical interest to consider just two types of transformation—unitary and antiunitary.

Other possibilities are related to either of the preceding via trivial phase changes.

The proof of this assertion is actually very difficult and will not be discussed further here (Gottfried 1966, 226-28).