تبهگنی کرامرز چیست؟

آیا تبهگنی کرامرز به تعداد الکترونها بستگی دارد؟

آیا میدان الکتریکی همواره سبب کاهش تبهگنی می شود؟

آیا همواره هامیلتونی با عملگر وارونی زمان جابه جا می شود؟

اگر هامیلتونی با عملگر وارونی زمان جا به جا شود ، آیا ویژه حالت هامیلتونی با ویژه حالت وارون شده زمانی آن معرف یک حالت هستند که به یک انرژی منجر می شوند؟

آیا میدان مغناطیسی خارجی می تواند تبهگنی کرامرز را از بین ببرد؟

Interactions with Electric and Magnetic Fields; Kramers Degeneracy

Consider charged particles in an external electric or magnetic field.

If we have only a static electric field interacting with the electric charge, the interaction part of the Hamiltonian is just

$$V(\mathbf{x}) = e\phi(\mathbf{x})$$

where $\phi(\mathbf{x})$ is the electrostatic potential.

Because $\phi(\mathbf{x})$ is a real function of the time-reversal even operator x, we have

$$[\Theta, H] = 0$$

Unlike the parity case, $[\Theta, H] = 0$ does not lead to an interesting conservation law.

The reason is that

$$\Theta U(t, t_0) \neq U(t, t_0)\Theta$$

even if $[\Theta, H] = 0$ holds, so our discussion following $[G, H] = 0$ of Section 4.1 breaks down.

As a result, there is no such thing as the "conservation of time-reversal quantum number."

As we already mentioned, requirement $[\Theta, H] = 0$ does, however, lead to a nontrivial phase restriction—the reality of a nondegenerate wave function for a spinless system [see $\mathrm{H}\Theta|n\rangle = \Theta\mathrm{H}|n\rangle$ and $\langle \mathbf{x}'|n\rangle = \langle \mathbf{x}'|n\rangle^*$].

Another far-reaching consequence of time-reversal invariance is the **Kramers degeneracy**.

Suppose H and Θ commute, and let $|n\rangle$ and $\Theta|n\rangle$ be the energy eigenket and its time-reversed state, respectively.

It is evident from $[\Theta, H] = 0$ that $|n\rangle$ and $\Theta|n\rangle$ belong to the same energy eigenvalue $E_n (H\Theta|n\rangle = \Theta H|n\rangle = E_n\Theta|n\rangle)$.

The question is, Does $|n\rangle$ represent the same state as $\Theta|n\rangle$?

If it does, $|n\rangle$ and $\Theta|n\rangle$ can differ at most by a phase factor. Hence,

$$\Theta|n\rangle = e^{i\delta}|n\rangle$$

Applying Θ again to $\Theta|n\rangle = e^{i\delta}$, we have $\Theta^2|n\rangle = \Theta e^{i\delta}|n\rangle = e^{-i\delta}e^{i\delta}|n\rangle$; hence $\Theta^2|n\rangle = |n\rangle$

But this relation is impossible for half-integer j systems, for which Θ^2 is always -1, so we are led to conclude that $|n\rangle$ and $\Theta|n\rangle$, which have the same energy, must correspond to distinct states—that is, there must be a degeneracy.

This means, for instance, that for a system composed of an odd number of electrons in an external electric field **E**, each energy level must be at least twofold degenerate no matter how complicated **E** may be.

Considerations along this line have interesting applications to electrons in crystals where odd-electron and even-electron systems exhibit very different behaviors.

Historically, Kramers inferred degeneracy of this kind by looking at explicit solutions of the Schrodinger equation; subsequently, Wigner pointed out that Kramers degeneracy is a consequence of time-reversal invariance.

Let us now turn to interactions with an external magnetic field.

The Hamiltonian H may then contain terms like

$$\mathbf{S} \cdot \mathbf{B}, \qquad \mathbf{p} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{p}, \qquad (\mathbf{B} = \nabla \times \mathbf{A})$$

where the magnetic field is to be regarded as external.

The operators S and p are odd under time reversal; these interaction terms therefore do lead to

$$\Theta H \neq H \Theta$$

As a trivial example, for a spin ½ system the spin-up state |+> and its time-reversed state |-> no longer have the same energy in the presence of an external magnetic field.

In general, Kramers degeneracy in a system containing an odd number of electrons can be lifted by applying an external magnetic field.

Notice that when we treat B as external, we do not change B under time reversal; this is because the atomic electron is viewed as a closed quantum-mechanical system to which we apply the time-reversal operator.

This should not be confused with our earlier remarks concerning the invariance of the Maxwell equations

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \quad \nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi \mathbf{j}}{c}, \quad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},$$

and the Lorentz force equation $\boldsymbol{F} = e[\boldsymbol{E} + \frac{1}{c}(\boldsymbol{v} \times \boldsymbol{B})$ under t →-t and the following transformations

$$\mathbf{E} \to \mathbf{E}, \qquad \mathbf{B} \to -\mathbf{B}, \qquad \rho \to \rho, \qquad \mathbf{j} \to -\mathbf{j}, \qquad \mathbf{v} \to -\mathbf{v}.$$

There we were to apply time reversal to the whole world, for example, even to the currents in the wire that produces the B field!