Quick Review

$$H = H_0 + V(t) \qquad H_0|n\rangle = E_n|n\rangle$$

$$|\alpha\rangle = \sum_n c_n(0)|n\rangle \qquad |\alpha, t_0 = 0; t\rangle = \sum_n c_n(t) e^{-iE_n t/\hbar}|n\rangle$$

$$i\hbar \frac{d}{dt} c_n(t) = \sum_m V_{nm} e^{i\omega_{nm} t} c_m(t) \equiv i\hbar \begin{pmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{c}_3 \\ \vdots \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12}e^{i\omega_{12}t} & & \cdots \\ V_{21}e^{i\omega_{21}t} & V_{22} & & \cdots \\ & & V_{33} & \cdots \\ \vdots & \vdots & & \ddots \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \end{pmatrix}$$

$$|\alpha, t_0; t\rangle_I = e^{iH_0 t/\hbar}|\alpha, t_0; t\rangle_S \qquad |\alpha\rangle_H = e^{+iHt/\hbar}|\alpha, t_0 = 0; t\rangle_S$$

$$A_I \equiv e^{iH_0 t/\hbar} A_S e^{-iH_0 t/\hbar} \qquad A_H = e^{iHt/\hbar} A_S e^{-iHt/\hbar}$$

$$V_I = e^{iH_0 t/\hbar} V e^{-iH_0 t/\hbar}$$

$$i\hbar \frac{\partial}{\partial t}|\alpha, t_0; t\rangle_I = V_I|\alpha, t_0; t\rangle_I$$

TIME-DEPENDENT PERTURBATION THEORY

Dyson Series

With the exception of a few problems like the two-level time-depen-time-dependent problem of the previous section, exact solutions to the differential equation for $c_n(t)$ are usually not available.

We must be content with approximate solutions to the differential equations of $c_n(t)$ obtained by perturbation expansion:

$$c_n(t) = c_n^{(0)} + c_n^{(1)} + c_n^{(2)} + \cdots$$

where $c_n^{(1)}, c_n^{(2)}, \ldots$ signify amplitudes of first order, second order, and so on in the strength parameter of the time-dependent potential. The iteration method used to solve this problem is similar to what we did in time-independent perturbation theory. If initially only the state i is populated, we approximate c_n on the right-hand side of differential equation (5.5.17) by $c_n^{(0)} = \delta_{ni}$ (independent of t) and relate it to the time derivative of $c_n^{(1)}$, integrate the differential equation to obtain $c_n^{(1)}$, plug $c_n^{(1)}$ into the right-hand side [of (5.5.17)] again to obtain the differential equation for $c_n^{(2)}$, and so on. This is how Dirac developed time-dependent perturbation theory in 1927.

$$i\hbar \begin{pmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{c}_3 \\ \vdots \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12}e^{i\omega_{12}t} & & \cdots \\ V_{21}e^{i\omega_{21}t} & V_{22} & & \cdots \\ & & V_{33} & \cdots \\ \vdots & \vdots & & \ddots \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \end{pmatrix}$$

Instead of working with $c_n(t)$, we propose to look at the time evolution operator $U_I(t, t_0)$ in the interaction picture, which we will define later. We obtain a perturbation expansion for $U_I(t, t_0)$, and at the very end we relate the matrix elements of U_I to $c_n(t)$. If we are interested only in solving simple problems in nonrelativistic quantum mechanics, all this might look superfluous; however, the operator formalism we develop is very powerful because it can immediately be applied to more-advanced problems, such as relativistic quantum field theory and many-body theory.

The time-evolution operator in the interaction picture is defined by

$$|\alpha, t_0; t\rangle_I = U_I(t, t_0)|\alpha, t_0; t_0\rangle_I$$

Differential equation for the state ket of the interaction picture is equivalent to

$$i\hbar \frac{d}{dt} U_I(t, t_0) = V_I(t) U_I(t, t_0) \quad \equiv \quad i\hbar \frac{\partial}{\partial t} |\alpha, t_0; t\rangle_I = V_I |\alpha, t_0; t\rangle_I$$

$$i\hbar\frac{\partial}{\partial t}|\alpha, t_0; t\rangle_I = V_I|\alpha, t_0; t\rangle_I$$

$$|\alpha, t_0; t\rangle_I = U_I(t, t_0)|\alpha, t_0; t_0\rangle_I$$

$$i\hbar\left[\frac{\partial}{\partial t}U_I(t, t_0)\right]|\alpha, t_0, t_0\rangle + 0 \times U_I(t, t_0) = V_I U_I(t, t_0)|\alpha, t_0, t_0\rangle$$

$$i\hbar\left[\frac{\partial}{\partial t}U_I(t, t_0)\right]|\alpha, t_0, t_0\rangle = V_I U_I(t, t_0)|\alpha, t_0, t_0\rangle$$

$$i\hbar\frac{\partial}{\partial t}U_I(t, t_0) = V_I U_I(t, t_0)$$

We must solve this operator differential equation subject to the initial condition

$$U_I(t,t_0)|_{t=t_0}=1 \qquad i\hbar\frac{d}{dt}U_I(t,t_0)=V_I(t)U_I(t,t_0)$$

First, let us note that the differential equation together with the initial condition is equivalent to the following integral equation:

$$U_I(t,t_0)=1-\frac{i}{\hbar}\int_{t_0}^{t}V_I(t')U_I(t',t_0)\,dt'$$

We can obtain an approximate solution to this equation by iteration:

$$U_I(t,t_0) = 1 - \frac{i}{\hbar}\int_{t_0}^{t} V_I(t')\,U_I(t',t_0)\,dt'$$

$$U_I(t,t_0) = 1 - \frac{i}{\hbar}\int_{t_0}^{t} V_I(t')\left[1 - \frac{i}{\hbar}\int_{t_0}^{t'} V_I(t'')\,U_I(t'',t_0)\,dt''\right]dt'$$

$$= 1 - \frac{i}{\hbar}\int_{t_0}^{t} dt'\,V_I(t') + \left(\frac{-i}{\hbar}\right)^2 \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt''\,V_I(t')V_I(t'')$$

$$+ \cdots + \left(\frac{-i}{\hbar}\right)^n \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \cdots$$

Dyson series

$$\times \int_{t_0}^{t^{(n-1)}} dt^{(n)} V_I(t')V_I(t'')\cdots V_I(t^{(n)})$$

$$+ \cdots .$$

Transition Probability

Once $U_I(t, t_0)$ is given, we can predict the time development of any state ket. For example, if the initial state at $t = 0$ is one of the energy eigenstates of H_0, then to obtain the initial state ket at a later time, all we need to do is multiply by $U_I(t,0)$:

$$|i, t_0 = 0; t\rangle_I = U_I(t,0)|i\rangle$$

$$= \sum_n |n\rangle\langle n|U_I(t,0)|i\rangle$$

In fact, $\langle n|U_I(t,0)|i\rangle$ is nothing more than what we called $c_n(t)$ earlier

$$|\alpha, t_0 = 0; t\rangle = \sum_n c_n(t)e^{-iE_nt/\hbar}|n\rangle$$

We earlier introduced the time-evolution operator $U(t,t_0)$ in the Schrodinger picture (see Section 2.2). Let us now explore the connection between $U(t, t_0)$ and $U_I(t\ t_0)$.

$$|\alpha, t_0; t\rangle_I = e^{iH_0 t/\hbar}|\alpha, t_0; t\rangle_S$$

$$= e^{iH_0 t/\hbar}U(t, t_0)|\alpha, t_0; t_0\rangle_S$$

$$= e^{iH_0 t/\hbar}U(t, t_0)e^{-iH_0 t_0/\hbar}|\alpha, t_0; t_0\rangle_I$$

$$|\alpha, t_0; t\rangle_I = U_I(t, t_0)|\alpha, t_0; t_0\rangle_I$$

$$U_I(t, t_0) = e^{iH_0 t/\hbar}U(t, t_0)e^{-iH_0 t_0/\hbar}$$

Let us now look at the matrix element of $U_I(t, t_0)$ between energy eigenstates of H_0:

$$\langle n|U_I(t, t_0)|i\rangle = e^{\iota(E_n t - E_i t_0)/\hbar}\langle n|U(t, t_0)|i\rangle$$

We recall from Section 2.2 that $\langle n|U(t, t_0)|i\rangle$ is defined to be the transition amplitude. Hence our $\langle n|U_I(t, t_0)|i\rangle$ here is not quite the same as the transition amplitude defined earlier. However, the transition *probability* defined as the square of the modulus of $\langle n|U(t, t_0)|i\rangle$ is the same as the analogous quantity in the interaction picture

$$|\langle n|U_I(t, t_0)|i\rangle|^2 = |\langle n|U(t, t_0)|i\rangle|^2$$

Parenthetically, we may remark that if the matrix elements of U_I are taken between initial and final states that are not energy eigenstates—for example, between $|a'\rangle$ and $|b'\rangle$ (eigenkets of A and B, respectively), where $[H_0, A] \neq 0$ and/or $[H_0, B] \neq 0$—we have, in general,

$$|\langle b'|U_I(t, t_0)|a'\rangle| \neq |\langle b'|U(t, t_0)|a'\rangle|,$$

$$|i, t_0; t_0\rangle_S = e^{-\imath E_i t_0/\hbar}|i\rangle$$

$$|i, t_0; t_0\rangle_I = |i\rangle$$

$$|i, t_0; t\rangle_I = U_I(t, t_0)|i\rangle$$

$$|\alpha, t_0, t_0\rangle_I = \sum_n |n\rangle\langle n|U_I(t, t_0)|i\rangle$$

$$U_I(t,t_0) = 1 - \frac{i}{\hbar}\int_{t_0}^{t} V_I(t')\left[1 - \frac{i}{\hbar}\int_{t_0}^{t'} V_I(t'')U_I(t'',t_0)\,dt''\right]dt'$$

$$= 1 - \frac{i}{\hbar}\int_{t_0}^{t} dt' V_I(t') + \left(\frac{-i}{\hbar}\right)^2 \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' V_I(t')V_I(t'')$$

$$+ \cdots + \left(\frac{-i}{\hbar}\right)^n \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \cdots$$

$$\times \int_{t_0}^{t^{(n-1)}} dt^{(n)} V_I(t')V_I(t'')\cdots V_I(t^{(n)})$$

$$+ \cdots . \qquad (3$$

$$e^{\imath(E_n - E_i)t/\hbar} = e^{\imath\omega_{ni}t}$$

If we compare it with:

$$|i, t_0; t\rangle_I = \sum_n c_n(t)|n\rangle$$

We have:

$$c_n(t) = \langle n|U_I(t, t_0)|i\rangle$$

$$c_n^{(0)}(t) = \delta_{ni} \qquad \text{(independent of } t\text{)}$$

$$c_n^{(1)}(t) = \frac{-i}{\hbar}\int_{t_0}^{t} \langle n|V_I(t')|i\rangle\, dt'$$

$$= \frac{-i}{\hbar}\int_{t_0}^{t} e^{\imath\omega_{ni}t'} V_{ni}(t')\, dt'$$

$$c_n^{(2)}(t) = \left(\frac{-i}{\hbar}\right)^2 \sum_m \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' e^{\imath\omega_{nm}t'} V_{nm}(t') e^{\imath\omega_{mi}t''} V_{mi}(t'')$$

The transition probability for $|i\rangle \to |n\rangle$ with $n \neq i$ is obtained by

$$P(i \to n) = |c_n^{(1)}(t) + c_n^{(2)}(t) + \cdots|^2.$$

Constant Perturbation

$$V(t) = \begin{cases} 0, & \text{for } t < 0 \\ V \quad \text{(independent of } t\text{)}, & \text{for } t \geq 0 \end{cases}$$

Even though the operator V has no explicit dependence on time, it is, in general, made up of operators like **x**, **p**, and **s**. Now suppose at $t = 0$, we have only $|i\rangle$. With t_0 taken to be zero, we obtain

$$c_n^{(0)} = c_n^{(0)}(0) = \delta_{in},$$

$$c_n^{(1)} = \frac{-i}{\hbar} V_{ni} \int_0^t e^{i\omega_{ni}t'}\, dt'$$

$$= \frac{V_{ni}}{E_n - E_i}\left(1 - e^{i\omega_{ni}t}\right)$$

$$|c_n^{(1)}|^2 = \frac{|V_{ni}|^2}{|E_n - E_i|^2}(2 - 2\cos\omega_{ni}t)$$

$$= \frac{4|V_{ni}|^2}{|E_n - E_i|^2}\sin^2\left[\frac{(E_n - E_i)t}{2\hbar}\right]$$

$$|c_n^{(1)}|^2 = \frac{|V_{ni}|^2}{|E_n - E_i|^2}(2 - 2\cos\omega_{ni}t)$$

$$= \frac{4|V_{ni}|^2}{|E_n - E_i|^2}\sin^2\left[\frac{(E_n - E_i)t}{2\hbar}\right]$$

The probability of finding $|n\rangle$ depends not only on $|V_{ni}|^2$ but also on the energy difference $E_n - E_i$, so let us try to see how (5.6.22) looks as a function of E_n. In practice, we are interested in this way of looking at (5.6.22) when there are many states with $E \sim E_n$ so that we can talk about a continuum of final states with nearly the same energy. To this end we define

$$\omega \equiv \frac{E_n - E_i}{\hbar}$$

and plot $4\sin^2(\omega t/2)/\omega^2$ as a function of ω for fixed t, the time interval during which the perturbation has been on; see Figure 5.6. We see that the height of the middle peak, centered around $\omega = 0$, is t^2 and the width is proportional to $1/t$. As t becomes large, $|c_n^{(1)}(t)|^2$ is appreciable only for those final states that satisfy

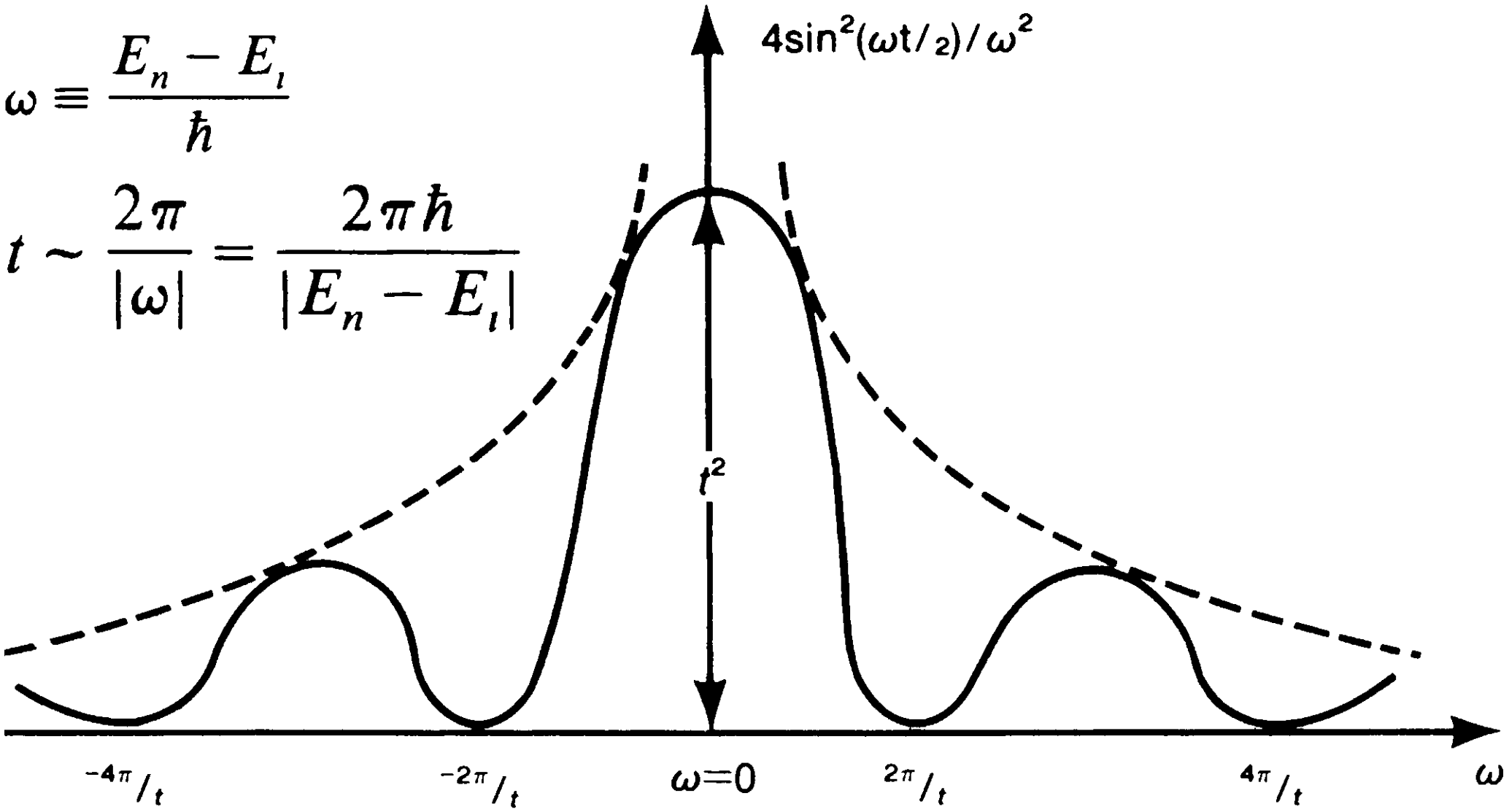
$$t \sim \frac{2\pi}{|\omega|} = \frac{2\pi\hbar}{|E_n - E_i|}$$

$$|c_n^{(1)}|^2 = \frac{|V_{ni}|^2}{|E_n - E_i|^2}(2 - 2\cos\omega_{ni}t)$$

$$= \frac{4|V_{ni}|^2}{|E_n - E_i|^2}\sin^2\left[\frac{(E_n - E_i)t}{2\hbar}\right]$$

$$\omega \equiv \frac{E_n - E_i}{\hbar}$$

$$t \sim \frac{2\pi}{|\omega|} = \frac{2\pi\hbar}{|E_n - E_i|}$$



If we call Δt the time interval during which the perturbation has been turned on, a transition with appreciable probability is possible only if

$$\Delta t \, \Delta E \sim \hbar, \tag{5.6.25}$$

where by ΔE we mean the energy change involved in a transition with appreciable probability. If Δt is small, we have a broader peak in Figure 5.6, and as a result we can tolerate a fair amount of energy *non*conservation. On the other hand, if the perturbation has been on for a very long time, we have a very narrow peak, and approximate energy conservation is required for a transition with appreciable probability. Note that this "uncertainty relation" is fundamentally different from the $x - p$ uncertainty relation of Section 1.6. There x and p are both observables. In contrast, time in nonrelativistic quantum mechanics is a parameter, not an observable.

For those transitions with exact energy conservation $E_n = E_i$, we have

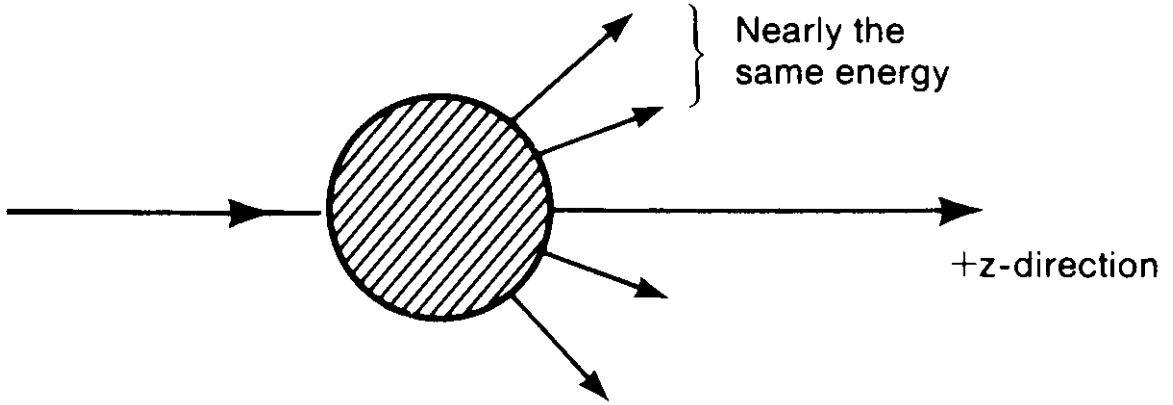
$$|c_n^{(1)}(t)|^2 = \frac{1}{\hbar^2}|V_{ni}|^2 t^2$$

$$|c_n^{(1)}|^2 = \frac{|V_{ni}|^2}{|E_n - E_i|^2}(2 - 2\cos\omega_{ni}t)$$

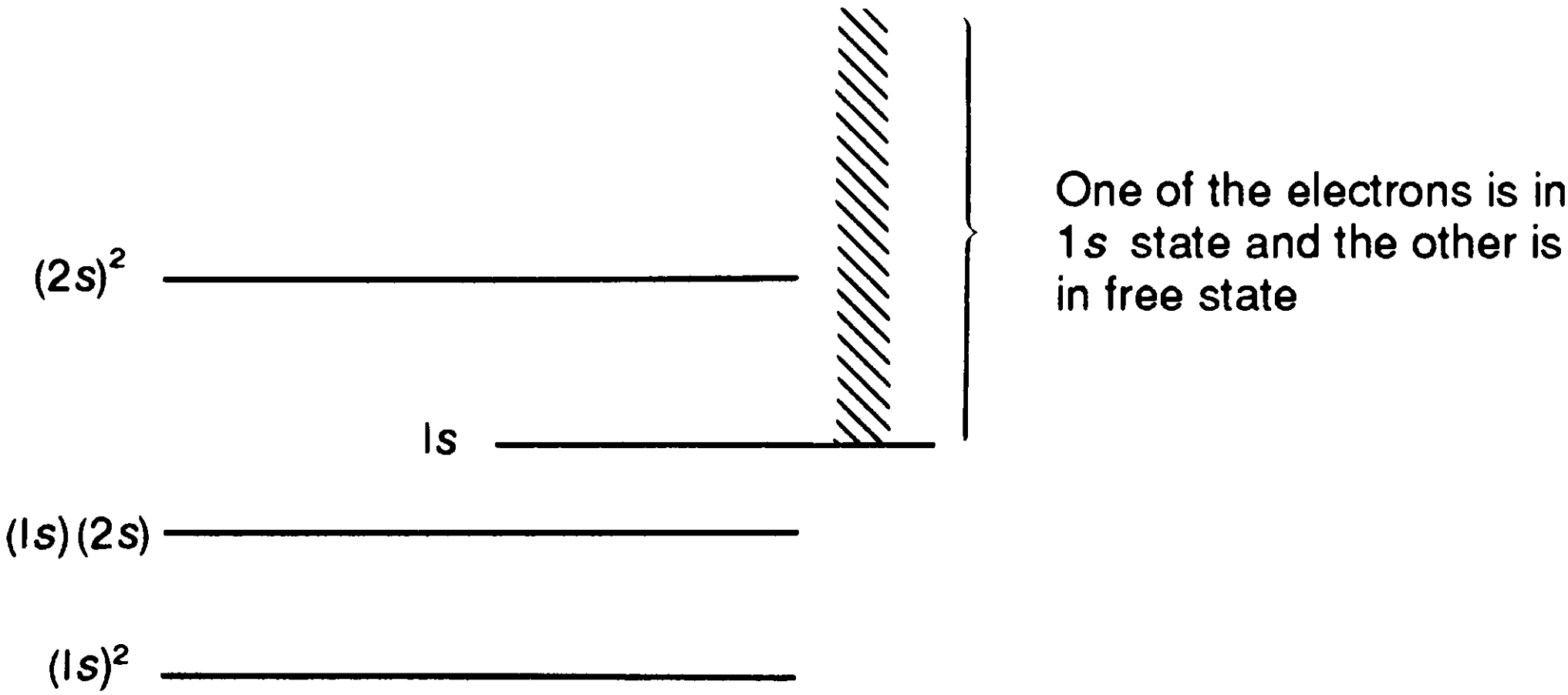
$$= \frac{4|V_{ni}|^2}{|E_n - E_i|^2}\sin^2\left[\frac{(E_n - E_i)t}{2\hbar}\right]$$

$$|c_n^{(1)}(t)|^2 = \frac{1}{\hbar^2}|V_{ni}|^2 t^2$$

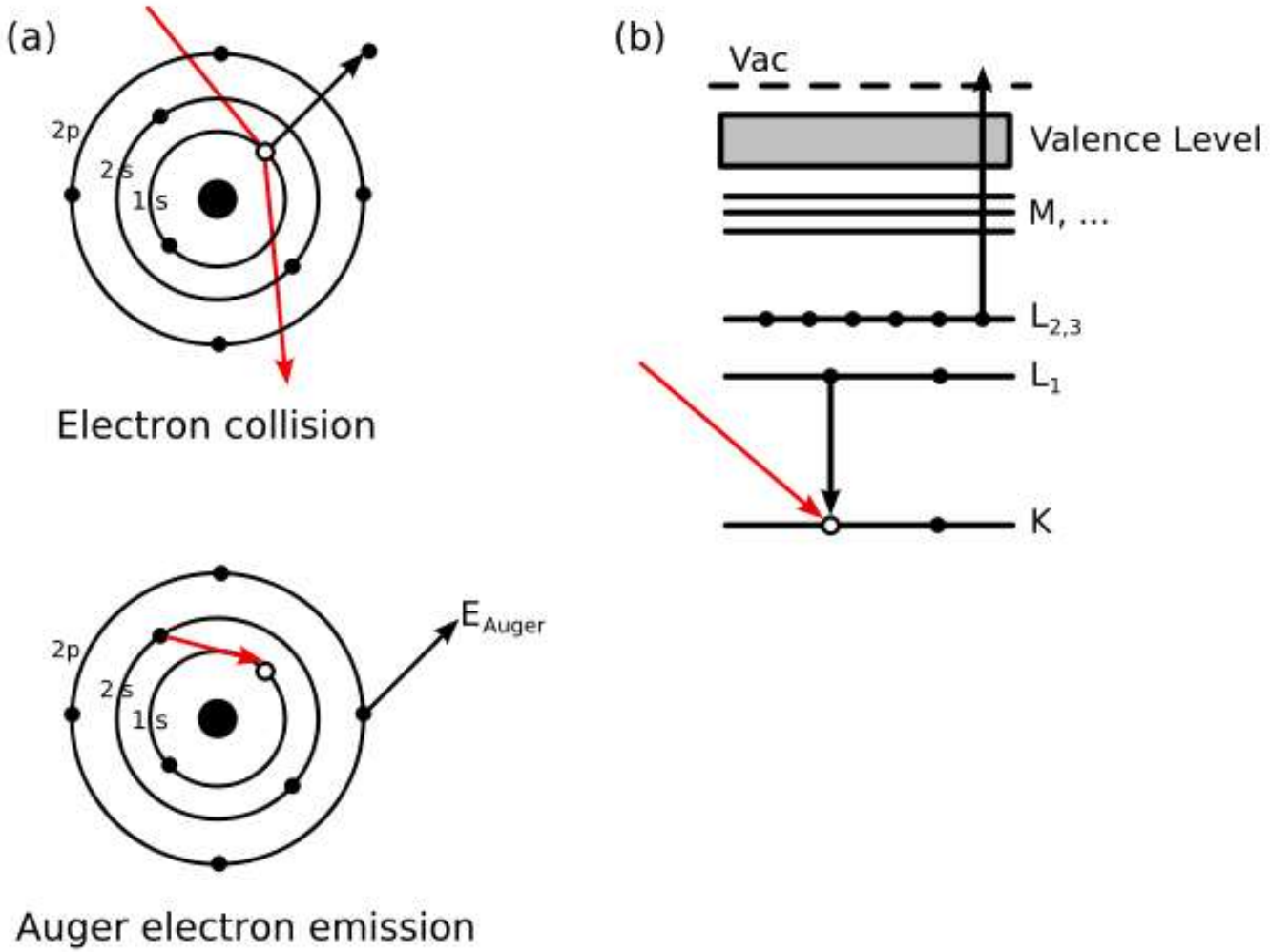
The probability of finding $|n\rangle$ after a time interval t is quadratic, *not* linear, in the time interval during which V has been on. This may appear intuitively unreasonable. There is no cause for alarm, however. In a realistic situation where our formalism is applicable, there is usually a group of final states, all with nearly the same energy as the energy of the initial state $|i\rangle$. In other words, a final state forms a continuous energy spectrum in the neighborhood of E_i. We give two examples along this line. Consider for instance, elastic scattering by some finite range potential (see Figure 5.7), which we will consider in detail in Chapter 7. The initial state is taken to be a plane wave state with its propagation direction oriented in the positive z-direction; the final state may also be a plane wave state of the same energy but with its propagation direction, in general, in a direction other than the positive z-direction.



Another example of interest is the de-excitation of an excited atomic state via the emission of an Auger electron. The simplest example is a helium atom. The initial state may be $(2s)^2$, where both the electrons are excited; the final state may be $(1s)$ (that is, one of the electrons still bound) of the He^+ ion, while the second electron escapes with a positive energy E; see Figure 5.8.



The **Auger effect** is a physical phenomenon in which the transition of an electron in an atom filling in an inner-shell vacancy causes the emission of another electron.[1] When a core electron is removed, leaving a vacancy, an electron from a higher energy level may fall into the vacancy, resulting in a release of energy. Although sometimes this energy is released in the form of an emitted photon, the energy can also be transferred to another electron, which is ejected from the atom. This second ejected electron is called an **Auger electron**,[2] after one of its discoverers, Pierre Victor Auger.


(a)
2p
2s
1s
Electron collision
2p
2s
1s
E Auger
Auger electron emission
(b)
Vac
Valence Level
M, ...
L2,3
L1
K

In such a case we are interested in the total probability—that is, the transition probabilities summed over final states with $E_n \sim E_t$

$$\sum_{n,\, E_n \simeq E_i} |c_n^{(1)}|^2$$

It is customary to define the density of final states as the number of states within energy interval $(E, E + dE)$ as

$$\rho(E)\, dE$$

$$\sum_{n,\, E_n \simeq E_i} |c_n^{(1)}|^2 \Rightarrow \int dE_n \rho(E_n) |c_n^{(1)}|^2$$

$$= 4 \int \sin^2 \left[\frac{(E_n - E_i)t}{2\hbar} \right] \frac{|V_{ni}|^2}{|E_n - E_i|^2} \rho(E_n)\, dE_n$$

$$\lim_{\alpha \to \infty} \frac{1}{\pi} \frac{\sin^2 \alpha x}{\alpha x^2} = \delta(x)$$

$$\lim_{t \to \infty} \frac{1}{|E_n - E_i|^2} \sin^2 \left[\frac{(E_n - E_i)t}{2\hbar} \right] = \frac{\pi t}{2\hbar} \delta(E_n - E_i)$$

It is now possible to take the average of $|V_{ni}|^2$ outside the integral sign and perform the integration with the δ-function:

$$\lim_{t \to \infty} \int dE_n \rho(E_n) |c_n^{(1)}(t)|^2 = \left(\frac{2\pi}{\hbar} \right) \overline{|V_{ni}|^2} \rho(E_n) t \Big|_{E_n \simeq E_i}$$

Thus the total transition probability *is* proportional to t for large values of t, which is quite reasonable. Notice that this linearity in t is a consequence of the fact that the total transition probability is proportional to the area under the peak of Figure 5.6, where the height varies as t^2 and the width varies as $1/t$.

It is conventional to consider the **transition rate**—that is, the transition probability per unit time. Expression (5.6.32) tells us that the total transition rate, defined by

$$\frac{d}{dt}\left(\sum_n |c_n^{(1)}|^2\right), \tag{5.6.33}$$

is constant in t for large t. Calling (5.6.33) $w_{i\to[n]}$, where $[n]$ stands for a group of final states with energy similar to i, we obtain

$$w_{i\to[n]} = \frac{2\pi}{\hbar}\overline{|V_{ni}|^2}\rho(E_n)_{E_n\approx E_i} \tag{5.6.34}$$

independent of t, provided the first-order time-dependent perturbation theory is valid. This formula is of great practical importance; it is called **Fermi's golden rule** even though the basic formalism of t-dependent perturbation theory is due to Dirac.* We sometimes write (5.6.34) as

$$w_{i\to n} = \left(\frac{2\pi}{\hbar}\right)|V_{ni}|^2\delta(E_n - E_i), \tag{5.6.35}$$

where it must be understood that this expression is integrated with $\int dE_n\rho(E_n)$.

Fermi's golden rule

From Wikipedia, the free encyclopedia

In quantum physics, **Fermi's golden rule** is a way to calculate the transition rate (probability of transition per unit time) from one energy eigenstate of a quantum system into a continuum of energy eigenstates, due to a perturbation.

We consider the system to begin in an eigenstate, $|i\rangle$, of a given Hamiltonian, H_0. We consider the effect of a (possibly time-dependent) perturbing Hamiltonian, H'. If H' is time-independent, the system goes only into those states in the continuum that have the same energy as the initial state. If H' is oscillating as a function of time with an angular frequency ω, the transition is into states with energies that differ by $\hbar\omega$ from the energy of the initial state. In both cases, the one-to-many transition probability per unit of time from the state $|i\rangle$ to a set of final states $|f\rangle$ is given, to first order in the perturbation, by

$$T_{i\to f} = \frac{2\pi}{\hbar} \left|\langle f|H'|i\rangle\right|^2 \rho,$$

where ρ is the density of final states (number of states per unit of energy) and $\langle f|H'|i\rangle$ is the matrix element (in bra-ket notation) of the perturbation H' between the final and initial states.This transition probability is also called decay probability and is related to mean lifetime.

We should also understand what is meant by $|V_{nl}|^2$. If the final states $|n\rangle$ form a quasi-continuum, the matrix elements V_{nl} are often similar if $|n\rangle$ are similar. However, it may happen that all energy eigenstates with the same E_n do not necessarily have similar matrix elements. Consider, for example, elastic scattering. The $|V_{nl}|^2$ that determines the scattering cross section may depend on the final momentum direction. In such a case the group of final states we should consider must have not only approximately the same energy, but they must also have approximately the same momentum direction. This point becomes clearer when we discuss the photoelectric effect.

Let us now look at the second-order term, still with the constant perturbation of (5.6.20). From (5.6.17) we have

$$c_n^{(2)} = \left(\frac{-i}{\hbar}\right)^2 \sum_m V_{nm} V_{mi} \int_0^t dt' \, e^{i\omega_{nm} t'} \int_0^{t'} dt'' \, e^{i\omega_{mi} t''}$$

$$= \frac{i}{\hbar} \sum_m \frac{V_{nm} V_{mi}}{E_m - E_i} \int_0^t \left(e^{i\omega_{ni} t'} - e^{i\omega_{nm} t'}\right) dt'. \tag{5.6.36}$$

The first term on the right-hand side has the same t-dependence as $c_n^{(1)}$ [see (5.6.21)]. If this were the only term, we could then repeat the same argument as before and conclude that as $t \to \infty$, the only important contribution arises from $E_n \simeq E_i$. Indeed, when E_m differs from E_n and E_i, the second contribution gives rise to a rapid oscillation, which does not give a contribution to the transition probability that grows with t.

With $c^{(1)}$ and $c^{(2)}$ together, we have

$$w_{i\to[n]} = \frac{2\pi}{\hbar}\overline{\left|V_{ni} + \sum_m \frac{V_{nm}V_{mi}}{E_i - E_m}\right|^2}\,\rho(E_n)\Bigg|_{E_n \simeq E_i}$$

The formula has the following physical interpretation. We visualize that the transition due to the second-order term takes place in two steps. First, $|i\rangle$ makes an energy nonconserving transition to $|m\rangle$; subsequently, $|m\rangle$ makes an energy nonconserving transition to $|n\rangle$, where between $|n\rangle$ and $|i\rangle$ there is overall energy conservation. Such energy nonconserving transitions are often called *virtual transitions*. Energy need not be conserved for those virtual transitions into (or from) virtual intermediate states. In contrast, the first-order term V_{ni} is often said to represent a direct energy-conserving "real" transition. A special treatment is needed if $V_{nm}V_{mi} \neq 0$ with $E_m \simeq E_i$. The best way to treat this is to use the slow-turn-on method $V \to e^{\eta t}V$, which we will discuss in Section 5.8 and Problem 31 of this chapter. The net result is to change the energy denominator in (5.6.37) as follows:

$$E_i - E_m \to E_i - E_m + i\varepsilon$$