1.3. Further contact between statistics and thermodynamics

In continuation of the preceding considerations, we now examine a more elaborate exchange between the sub-systems A_1 and A_2. If we assume that the wall separating the two sub-systems is movable as well as conducting, then the respective volumes V_1 and V_2 (of sub-systems A_1 and A_2) also become variable; indeed, the total volume $V^{(0)}(=V_1+V_2)$ remains constant, so that effectively we have only one more independent variable. Of course, the wall is still assumed to be impenetrable to particles, so the numbers N_1 and N_2 remain fixed. Arguing as before, the state of equilibrium for the composite system $A^{(0)}$ will obtain when the number $\Omega^{(0)}(V^{(0)}, E^{(0)}; V_1, E_1)$ attains its largest value, i.e. when not only

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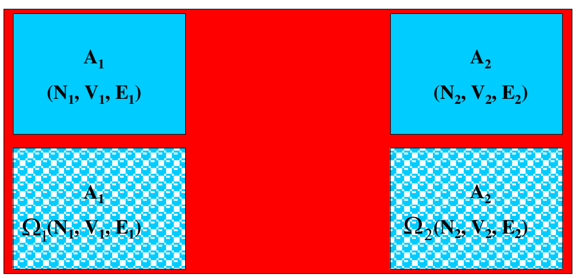
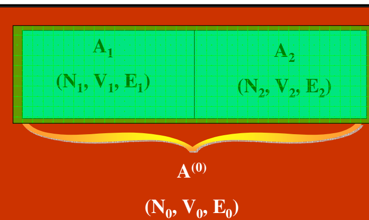
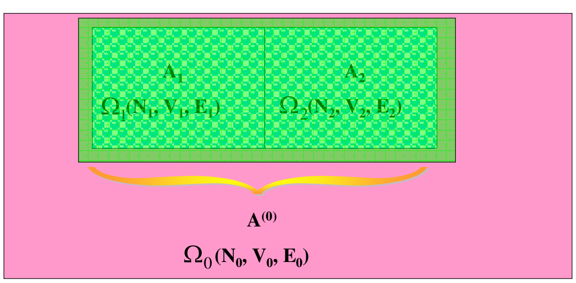




$$N^{(0)} = N_1 + N_2 = \text{Const} \quad \text{and} \quad N_1, N_2 = \text{Const}$$

$$V^{(0)} = V_1 + V_2 = \text{Const} \;\Rightarrow dV^{(0)} = dV_1 + dV_2 = 0 \;\Rightarrow dV_1 = -dV_2$$

$$E^{(0)} = E_1 + E_2 = \text{Const} \;\Rightarrow dE^{(0)} = dE_1 + dE_2 = 0 \;\Rightarrow dE_1 = -dE_2$$



$$\mathbf{\Omega^{(0)}(E_1, V_1, \cancel{N_1}; E_2, V_2, \cancel{N_2}) =}$$

$$\mathbf{\Omega^{(0)}(E_1, V_1; \cancel{E_2}, \cancel{V_2}) = \Omega_1(E_1, V_1)\Omega_2(E_2, V_2)}$$

$$\mathbf{= \Omega_1(E_1, V_1)\Omega_2(\cancel{E^{(0)}} - E_1, \cancel{V^{(0)}} - V_1)}$$

$$\mathbf{= \Omega_1(E_1, V_1)\Omega_2(E_1, V_1)}$$

$$\mathbf{= \Omega^{(0)}(E_1, V_1)}$$

Clearly, the number $\Omega^{(0)}$ itself varies with E_1 and V_1.

تعداد آن دسته از حالتهای دستگاه $A^{(0)}$ که در آن حالتها انرژی دستگاه A_1 برابر E_1 و حجم آن برابر V_1 است.

$$P(E_1,V_1)=\frac{\Omega^{(0)}(E_1,V_1)}{\Omega^{(0)}_{total}}$$

تعداد کل حالتهای قابل دسترس دستگاه $A^{(0)}$ مستقل از انرژی و حجم دستگاه A_1.

$$P(E_1,V_1)=\frac{\Omega^{(0)}(E_1,V_1)}{\Omega^{(0)}_{total}}=C\Omega^{(0)}(E_1,V_1)\qquad ;C^{-1}=\Omega^{(0)}_{total}$$

$$=C\,\Omega_1(E_1,V_1)\,\Omega_2(E_1,V_1)$$

$$=C\Omega_1(E_1,V_1)\,\Omega_2\left(E^{(0)}-E_1,V^{(0)}-V_1\right)$$

$$=C\Omega_1(E_1,V_1)\,\Omega_2(E_2,V_2)$$

$$Lnp(E_1,V_1)=LnC+Ln\Omega_1(E_1,V_1)+Ln\Omega_2(E_2,V_2)$$

$$dLnp(E_1,V_1)=0+\frac{\partial Ln\Omega_1(E_1,V_1)}{\partial E_1}\bigg|_{V_1,N_1,\bar{E}_1}dE_1+\frac{\partial Ln\Omega_1(E_1,V_1)}{\partial V_1}\bigg|_{\bar{V}_1,N_1,E_1}dV_1+$$

$$+\frac{\partial Ln\Omega_2(E_2,V_2)}{\partial E_2}\bigg|_{N_2,V_2,\bar{E}_2}dE_2+\frac{\partial Ln\Omega_2(E_2,V_2)}{\partial V_2}\bigg|_{N_2,\bar{V}_2,E_2}dV_2=0$$

$$dE^{(0)}=0 \Rightarrow dE_1+dE_2=0 \Rightarrow dE_1=-dE_2$$

$$dV^{(0)}=0 \Rightarrow dV_1+dV_2=0 \Rightarrow dV_1=-dV_2$$

$$\left\{\frac{\partial Ln\Omega_1(E_1,V_1)}{\partial E_1}\bigg|_{N_1,V_1,\bar{E}_1}-\frac{\partial Ln\Omega_2(E_2,V_2)}{\partial E_2}\bigg|_{N_2,V_2,\bar{E}_2}\right\}dE_1+$$

$$+\left\{\frac{\partial Ln\Omega_1(E_1,V_1)}{\partial V_1}\bigg|_{N_1,E_1,\bar{V}_1}-\frac{\partial Ln\Omega_2(E_2,V_2)}{\partial V_2}\bigg|_{N_2,E_2,\bar{V}_2}\right\}dV_1=0$$

$$\beta=\frac{\partial Ln\Omega(E,V)}{\partial E}\bigg|_{N,V,\bar{E}}$$

$$\eta=\frac{\partial Ln\Omega(E,V)}{\partial V}\bigg|_{N,E,\bar{V}}$$

$$\left\{\frac{\partial Ln\Omega_1(E_1,V_1)}{\partial E_1}\bigg|_{N_1,V_1,\bar{E}_1}-\frac{\partial Ln\Omega_2(E_2,V_2)}{\partial E_2}\bigg|_{N_2,V_2,\bar{E}_2}\right\}dE_1+$$

$$+\left\{\frac{\partial Ln\Omega_1(E_1,V_1)}{\partial V_1}\bigg|_{N_1,E_1,\bar{V}_1}-\frac{\partial Ln\Omega_2(E_2,V_2)}{\partial V_2}\bigg|_{N_2,E_2,\bar{V}_2}\right\}dV_1\quad 0$$

$$\{\beta_1-\beta_2\}dE_1+\{\eta_1-\eta_2\}dV_1=0$$

$$\beta_1=\beta_2 \qquad\qquad \eta_1=\eta_2$$

بررسی مسأله از دیدگاه ترمودینامیکی

A_1 (N_1, V_1, E_1)	A_2 (N_2, V_2, E_2)

$A^{(0)}$

(N_0, V_0, E_0)

$$E^{(0)}=\sum_{\alpha=1}^{2}E_\alpha=E_1+E_2=Const. \Rightarrow \Delta E^{(0)}=0 \Rightarrow \Delta E_1=-\Delta E_2$$

$$V^{(0)}=\sum_{\alpha=1}^{2}V_\alpha=V_1+V_2=Const. \Rightarrow \Delta V^{(0)}=0 \Rightarrow \Delta V_1=-\Delta V_2$$

$$S^{(0)}=\sum_{\alpha=1}^{2}S_\alpha=S_1+S_2$$

آنتروپی را حول نقطه تعادل بسط تیلور می دهیم:

$$S^{(0)}=S^{(0)}\big|_0+\sum_{\alpha=1}^{2}\frac{\partial S_\alpha}{\partial E_\alpha}\bigg|_{N_\alpha,V_\alpha}\Delta E_\alpha+\sum_{\alpha=1}^{2}\frac{\partial S_\alpha}{\partial V_\alpha}\bigg|_{N_\alpha,E_\alpha}\Delta V_\alpha$$

$$S^{(0)}-S^{(0)}\big|_0=\Delta S^{(0)}$$

$$\Delta S^{(0)}=\frac{\partial S_1}{\partial E_1}\bigg|_{N_1,V_1}\Delta E_1+\frac{\partial S_2}{\partial E_2}\bigg|_{N_2,V_2}\Delta E_2+\frac{\partial S_1}{\partial V_1}\bigg|_{N_1,E_1}\Delta V_1+\frac{\partial S_2}{\partial V_2}\bigg|_{N_2,E_2}\Delta V_2+\ldots$$

$$\Delta S^{(0)}=\left\{\frac{\partial S_1}{\partial E_1}\bigg|_{N_1,V_1}-\frac{\partial S_2}{\partial E_2}\bigg|_{N_2,V_2}\right\}\Delta E_1+\left\{\frac{\partial S_1}{\partial V_1}\bigg|_{N_1,E_1}-\frac{\partial S_2}{\partial V_2}\bigg|_{N_2,E_2}\right\}\Delta V_1+\ldots$$

$$\langle q\rangle \Rightarrow \Delta S>0$$

$$\Delta S^{(0)}>0$$

$$\frac{\partial S_1}{\partial E_1}\bigg|_{N_1,V_1}-\frac{\partial S_2}{\partial E_2}\bigg|_{N_2,V_2}=0 \Rightarrow ?$$

$$\frac{\partial S_1}{\partial V_1}\bigg|_{N_1,V_1}-\frac{\partial S_2}{\partial V_2}\bigg|_{N_2,V_2}=0 \Rightarrow ?$$

$$\left.\begin{array}{l} dE = dQ - PdV + \mu dN \\ dQ = TdS \end{array}\right\} \Rightarrow dE = TdS - PdV + \mu dN$$

$$\left.\frac{\partial S}{\partial E}\right|_{V,N} = \frac{1}{T} \quad \& \quad \left.\frac{\partial S}{\partial V}\right|_{E,N} = \frac{P}{T}$$

$\Delta S^{(0)} > 0$

$$\left.\frac{\partial S_1}{\partial E_1}\right|_{N_1,V_1} - \left.\frac{\partial S_2}{\partial E_2}\right|_{N_2,V_2} = 0 \Rightarrow T_1 = T_2$$

$$\left.\frac{\partial S_1}{\partial V_1}\right|_{N_1,V_1} - \left.\frac{\partial S_2}{\partial V_2}\right|_{N_2,V_2} = 0 \Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\left.\frac{\partial \mathrm{Ln}\Omega_1(E_1)}{\partial E_1}\right|_{N_1,V_1,\bar{E}_1} = \left.\frac{\partial \mathrm{Ln}\Omega_2(E_2)}{\partial E_2}\right|_{N_2,V_2,\bar{E}_2}$$

آماری

$$\beta = \left.\frac{\partial \mathrm{Ln}\Omega(E)}{\partial E}\right|_{N,V,\bar{E}}$$

ترمودینامیک

$$\left.\frac{\partial S_1}{\partial E_1}\right|_{N_1,V_1} = \left.\frac{\partial S_2}{\partial E_2}\right|_{N_2,V_2}$$

$$\left.\frac{\partial S}{\partial E}\right|_{V,N} = \frac{1}{T}$$

$$\left.\frac{\Delta(\mathrm{Ln}\Omega(E))}{\Delta S}\right|_{E} = \beta T = \mathrm{const.}$$

$$\left.\frac{\partial \mathrm{Ln}\Omega_1(E_1,V_1)}{\partial V_1}\right|_{N_1,E_1,\bar{V}_1} = \left.\frac{\partial \mathrm{Ln}\Omega_2(E_2,V_2)}{\partial V_2}\right|_{N_2,E_2,\bar{V}_2}$$

آماری

$$\eta = \left.\frac{\partial \mathrm{Ln}\Omega(E)}{\partial V}\right|_{N,E,\bar{V}}$$

ترمودینامیک

$$\left.\frac{\partial S_1}{\partial V_1}\right|_{N_1,E_1} = \left.\frac{\partial S_2}{\partial V_2}\right|_{N_2,E_2}$$

$$\left.\frac{\partial S}{\partial V}\right|_{E,N} = \frac{P}{T}$$

$$\left.\frac{\Delta(\mathrm{Ln}\Omega(E,V))}{\Delta S}\right|_{E} = \frac{\eta T}{P} = \mathrm{const.}$$

$$\beta = \frac{1}{kT}$$

$$\left.\begin{array}{l} \eta = \left.\dfrac{\partial \mathrm{Ln}\Omega(N,V,E)}{\partial V}\right|_{N,E} \\ \left.\dfrac{\partial S}{\partial V}\right|_{N,E} = \dfrac{P}{T} \\ S = k\mathrm{Ln}\Omega(N,V,E) \end{array}\right\} \Rightarrow \eta = \frac{P}{kT}$$

Similarly, if A_1 and A_2 came into contact through a wall which allowed an exchange of particles as well, the conditions for equilibrium would be further augmented by the equality of the parameter ζ_1 of sub-system A_1 and the parameter ζ_2 of sub-system A_2 where, by definition,

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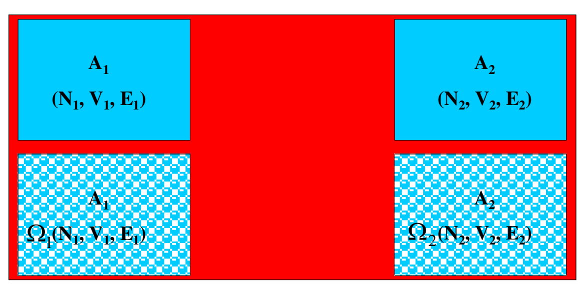
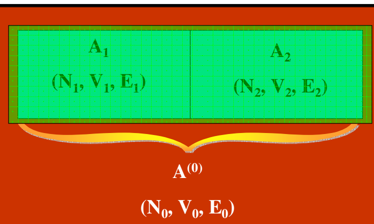
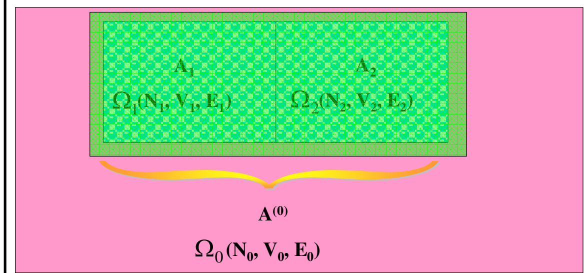




$$N^{(0)} = N_1 + N_2 = \text{Const} \Rightarrow dN^{(0)} = dN_1 + dN_2 = 0 \Rightarrow dN_1 = -dN_2$$

$$V^{(0)} = V_1 + V_2 = \text{Const} \Rightarrow dV^{(0)} = dV_1 + dV_2 = 0 \Rightarrow dV_1 = -dV_2$$

$$E^{(0)} = E_1 + E_2 = \text{Const} \Rightarrow dE^{(0)} = dE_1 + dE_2 = 0 \Rightarrow dE_1 = -dE_2$$



$$\Omega^{(0)}(E_1, V_1, N_1; \cancel{E_2}, \cancel{V_2}, \cancel{N_2}) =$$

$$= \Omega_1(N_1, E_1, V_1)\Omega_2(N_2, E_2, V_2)$$

$$= \Omega_1(N_1, E_1, V_1)\Omega_2(\cancel{N^{(0)}} - N_1, \cancel{E^{(0)}} - E_1, \cancel{V^{(0)}} - V_1)$$

$$= \Omega_1(N_1, E_1, V_1)\Omega_2(N_1, E_1, V_1)$$

$$= \Omega^{(0)}(N_1, E_1, V_1)$$

Clearly, the number $\Omega^{(0)}$ itself varies with N_1, E_1 and V_1.

تعداد آن دسته از حالتهای دستگاه $A^{(0)}$ که در آن حالتها انرژی دستگاه A_1 برابر E_1 ، حجم آن برابر V_1 و تعداد ذرات آن برابر N_1 است.

$$P(E_1, V_1) = \frac{\Omega^{(0)}(N_1, E_1, V_1)}{\Omega^{(0)}_{total}}$$

تعداد کل حالتهای قابل دسترس دستگاه $A^{(0)}$ مستقل از انرژی، حجم و تعداد ذرات دستگاه A_1.

$$P(N_1, E_1, V_1) = \frac{\Omega^{(0)}(N_1, E_1, V_1)}{\Omega^{(0)}_{total}} = C\Omega^{(0)}(N_1, E_1, V_1) \quad ; C^{-1} = \Omega^{(0)}_{total}$$

$$= C\,\Omega_1(N_1, E_1, V_1)\,\Omega_2(N_1, E_1, V_1)$$

$$= C\Omega_1(N_1, E_1, V_1)\,\Omega_2\left(N^{(0)} - N_1, E^{(0)} - E_1, V^{(0)} - V_1\right)$$

$$= C\Omega_1(N_1, E_1, V_1)\,\Omega_2(N_2, E_2, V_2)$$

$$\ln p(N_1, E_1, V_1) = \ln C + \ln\Omega_1(N_1, E_1, V_1) + \ln\Omega_2(N_2, E_2, V_2)$$

$$\begin{aligned}
d\ln p(N_1, E_1, V_1) = 0 &+ \left.\frac{\partial \ln\Omega_1(N_1, E_1, V_1)}{\partial E_1}\right|_{\bar{E}_1, V_1, N_1} dE_1 + \\
&+ \left.\frac{\partial \ln\Omega_1(N_1, E_1, V_1)}{\partial V_1}\right|_{\bar{V}_1, E_1, N_1} dV_1 + \\
&+ \left.\frac{\partial \ln\Omega_1(N_1, E_1, V_1)}{\partial N_1}\right|_{\bar{N}_1, E_1, V_1} dN_1 + \\
&+ \left.\frac{\partial \ln\Omega_2(N_1, E_1, V_1)}{\partial E_2}\right|_{\bar{E}_2, V_2, N_2} dE_2 + \\
&+ \left.\frac{\partial \ln\Omega_2(N_2, E_2, V_2)}{\partial V_2}\right|_{\bar{V}_2, E_2, N_2} dV_2 + \\
&+ \left.\frac{\partial \ln\Omega_2(N_2, E_2, V_2)}{\partial N_2}\right|_{\bar{N}_2, E_2, V_2} dN_2 \\
&= 0
\end{aligned}$$

$$dE^{(0)} = 0 \Longrightarrow dE_1 + dE_2 = 0 \Longrightarrow dE_1 = -dE_2$$

$$dV^{(0)} = 0 \Longrightarrow dV_1 + dV_2 = 0 \Longrightarrow dV_1 = -dV_2$$

$$dN^{(0)} = 0 \Longrightarrow dN_1 + dN_2 = 0 \Longrightarrow dN_1 = -dN_2$$

$$\begin{aligned}
&\left\{\left.\frac{\partial \ln\Omega_1(N_1, E_1, V_1)}{\partial E_1}\right|_{N_1, V_1, \bar{E}_1} - \left.\frac{\partial \ln\Omega_2(N_2, E_2, V_2)}{\partial E_2}\right|_{N_2, V_2, \bar{E}_2}\right\} dE_1 + \\
&+ \left\{\left.\frac{\partial \ln\Omega_1(N_1, E_1, V_1)}{\partial V_1}\right|_{N_1, E_1, \bar{V}_1} - \left.\frac{\partial \ln\Omega_2(N_2, E_2, V_2)}{\partial V_2}\right|_{N_2, E_2, \bar{V}_2}\right\} dV_1 + \\
&+ \left\{\left.\frac{\partial \ln\Omega_1(N_1, E_1, V_1)}{\partial N_1}\right|_{V_1, E_1, \bar{N}_1} - \left.\frac{\partial \ln\Omega_2(N_2, E_2, V_2)}{\partial N_2}\right|_{V_2, E_2, \bar{N}_2}\right\} dN_1 = \\
&= 0
\end{aligned}$$

بررسی مسأله از دیدگاه ترمودینامیکی

A_1	A_2
(N_1, V_1, E_1)	(N_2, V_2, E_2)

$$\underbrace{\hspace{6em}}_{\displaystyle A^{(0)}}$$

(N_0, V_0, E_0)

$$E^{(0)} = \sum_{\alpha=1}^{2} E_\alpha = E_1 + E_2 = Const. \Rightarrow \Delta E^{(0)} = 0 \Rightarrow \Delta E_1 = -\Delta E_2$$

$$V^{(0)} = \sum_{\alpha=1}^{2} V_\alpha = V_1 + V_2 = Const. \Rightarrow \Delta V^{(0)} = 0 \Rightarrow \Delta V_1 = -\Delta V_2$$

$$N^{(0)} = \sum_{\alpha=1}^{2} N_\alpha = N_1 + N_2 = Const. \Rightarrow \Delta N^{(0)} = 0 \Rightarrow \Delta N_1 = -\Delta N_2$$

$$\beta = \frac{\partial Ln\Omega(N,E,V)}{\partial E}\Big|_{N,V,\bar{E}} \qquad \eta = \frac{\partial Ln\Omega(N,E,V)}{\partial V}\Big|_{N,E,\bar{V}}$$

$$\xi = \frac{\partial Ln\Omega(N,E,V)}{\partial N}\Big|_{V,E,\bar{N}}$$

$$\left\{\frac{\partial Ln\Omega_1(N_1,E_1,V_1)}{\partial E_1}\Big|_{N_1,V_1,\bar{E}_1} - \frac{\partial Ln\Omega_2(N_2,E_2,V_2)}{\partial E_2}\Big|_{N_2,V_2,\bar{E}_2}\right\}dE_1 +$$

$$+\left\{\frac{\partial Ln\Omega_1(N_1,E_1,V_1)}{\partial V_1}\Big|_{N_1,E_1,\bar{V}_1} - \frac{\partial Ln\Omega_2(N_2,E_2,V_2)}{\partial V_2}\Big|_{N_2,E_2,\bar{V}_2}\right\}dV_1 +$$

$$+\left\{\frac{\partial Ln\Omega_1(N_1,E_1,V_1)}{\partial N_1}\Big|_{V_1,E_1,\bar{N}_1} - \frac{\partial Ln\Omega_2(N_2,E_2,V_2)}{\partial N_2}\Big|_{V_2,E_2,\bar{N}_2}\right\}dN_1 = 0$$

$$\{\beta_1 - \beta_2\}dE_1 + \{\eta_1 - \eta_2\}dV_1 + \{\xi_1 - \xi_2\}dN_1 = 0$$

$$\beta_1 = \beta_2 \qquad \eta_1 = \eta_2 \qquad \xi_1 = \xi_2$$

$$\langle q \rangle \Rightarrow \Delta S > 0$$

$$\Delta S^{(0)} > 0$$

$$\frac{\partial S_1}{\partial E_1}\Big|_{N_1,V_1} - \frac{\partial S_2}{\partial E_2}\Big|_{N_2,V_2} = 0 \Rightarrow ?$$

$$\frac{\partial S_1}{\partial V_1}\Big|_{N_1,V_1} - \frac{\partial S_2}{\partial V_2}\Big|_{N_2,V_2} = 0 \Rightarrow ?$$

$$\frac{\partial S_1}{\partial N_1}\Big|_{E_1,V_1} - \frac{\partial S_2}{\partial V_2}\Big|_{E_2,V_2} = 0 \Rightarrow ?$$

$$S^{(0)} = \sum_{\alpha=1}^{2} s_\alpha = s_1 + s_2$$

آنتروپی را حول نقطه تعادل بسط تیلور می دهیم:

$$S^{(0)} = S^{(0)}\Big|_0 + \sum_{\alpha=1}^{2}\frac{\partial S_\alpha}{\partial E_\alpha}\Big|_{N_\alpha,V_\alpha}\Delta E_\alpha + \sum_{\alpha=1}^{2}\frac{\partial S_\alpha}{\partial V_\alpha}\Big|_{N_\alpha,E_\alpha}\Delta V_\alpha + \sum_{\alpha=1}^{2}\frac{\partial S_\alpha}{\partial N_\alpha}\Big|_{V_\alpha,E_\alpha}\Delta N_\alpha$$

$$S^{(0)} - S^{(0)}\Big|_0 = \Delta S^{(0)}$$

$$\Delta S^{(0)} = \frac{\partial S_1}{\partial E_1}\Big|_{N_1,V_1}\Delta E_1 + \frac{\partial S_2}{\partial E_2}\Big|_{N_2,V_2}\Delta E_2 + \frac{\partial S_1}{\partial V_1}\Big|_{N_1,E_1}\Delta V_1 + \frac{\partial S_2}{\partial V_2}\Big|_{N_2,E_2}\Delta V_2 +$$

$$+\frac{\partial S_1}{\partial N_1}\Big|_{V_1,E_1}\Delta N_1 + \frac{\partial S_2}{\partial N_2}\Big|_{V_2,E_2}\Delta N_2 + \dots$$

$$\Delta S^{(0)} = \left\{\frac{\partial S_1}{\partial E_1}\Big|_{N_1,V_1} - \frac{\partial S_2}{\partial E_2}\Big|_{N_2,V_2}\right\}\Delta E_1 + \left\{\frac{\partial S_1}{\partial V_1}\Big|_{N_1,E_1} - \frac{\partial S_2}{\partial V_2}\Big|_{N_2,E_2}\right\}\Delta V_1 +$$

$$+\left\{\frac{\partial S_1}{\partial N_1}\Big|_{V_1,E_1} - \frac{\partial S_2}{\partial N_2}\Big|_{V_2,E_2}\right\}\Delta N_1 + \dots$$

$$\frac{\partial Ln\Omega_1(E_1)}{\partial E_1}\Big|_{N_1,V_1,\bar{E}_1} = \frac{\partial Ln\Omega_2(E_2)}{\partial E_2}\Big|_{N_2,V_2,\bar{E}_2}$$

آماری

$$\beta = \frac{\partial Ln\Omega(E)}{\partial E}\Big|_{N,V,\bar{E}}$$

ترمودینامیک

$$\frac{\partial S_1}{\partial E_1}\Big|_{N_1,V_1} = \frac{\partial S_2}{\partial E_2}\Big|_{N_2,V_2}$$

$$\frac{\partial S}{\partial E}\Big|_{V,N} = \frac{1}{T}$$

$$\frac{\Delta(Ln\Omega(E))}{\Delta S}\Big|_{E} = \beta T = const.$$

$$\left.\begin{array}{l} dE = dQ - PdV + \mu dN \\ dQ = TdS \end{array}\right\} \Rightarrow dE = TdS - PdV + \mu dN$$

$$\frac{\partial S}{\partial E}\Big|_{V,N} = \frac{1}{T} \quad \& \quad \frac{\partial S}{\partial V}\Big|_{E,N} = \frac{P}{T} \quad \& \quad \frac{\partial S}{\partial N}\Big|_{E,V} = -\frac{\mu}{T}$$

$$\Delta S^{(0)} > 0$$

$$\frac{\partial S_1}{\partial E_1}\Big|_{N_1,V_1} - \frac{\partial S_2}{\partial E_2}\Big|_{N_2,V_2} = 0 \Rightarrow T_1 = T_2$$

$$\frac{\partial S_1}{\partial V_1}\Big|_{N_1,V_1} - \frac{\partial S_2}{\partial V_2}\Big|_{N_2,V_2} = 0 \Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{\partial S_1}{\partial N_1}\Big|_{E_1,V_1} - \frac{\partial S_2}{\partial V_2}\Big|_{E_2,V_2} = 0 \Rightarrow \frac{\mu_1}{T_1} = \frac{\mu_2}{T_2}$$

$$\left.\frac{\partial \mathrm{Ln}\Omega_1(E_1,V_1)}{\partial V_1}\right|_{N_1,E_1,\bar{V}_1}=\left.\frac{\partial \mathrm{Ln}\Omega_2(E_2,V_2)}{\partial V_2}\right|_{N_2,E_2,\bar{V}_2}$$

آماری

$$\eta=\left.\frac{\partial \mathrm{Ln}\Omega(V,E)}{\partial V}\right|_{N,E,\bar{V}}$$

ترمودینامیک

$$\left.\frac{\partial S_1}{\partial V_1}\right|_{N_1,E_1}=\left.\frac{\partial S_2}{\partial V_2}\right|_{N_2,E_2}$$

$$\left.\frac{\partial S}{\partial V}\right|_{E,N}=\frac{P}{T}$$

$$\left.\frac{\Delta(\mathrm{Ln}\Omega(E,V))}{\Delta S}\right|_{\bar{E}}=\frac{\eta T}{P}=\text{const.}$$

$$\left.\frac{\partial \mathrm{Ln}\Omega_1(N_1,E_1,V_1)}{\partial N_1}\right|_{V_1,E_1,\bar{N}_1}=\left.\frac{\partial \mathrm{Ln}\Omega_2(N_2,E_2,V_2)}{\partial N_2}\right|_{V_2,E_2,\bar{N}_2}$$

آماری

$$\xi=\left.\frac{\partial \mathrm{Ln}\Omega(N,V,E)}{\partial N}\right|_{V,E,\bar{N}}$$

ترمودینامیک

$$\left.\frac{\partial S_1}{\partial N_1}\right|_{V_1,E_1}=\left.\frac{\partial S_2}{\partial N_2}\right|_{V_2,E_2}$$

$$\left.\frac{\partial S}{\partial N}\right|_{E,N}=-\frac{\mu}{T}$$

$$\left.\frac{\Delta(\mathrm{Ln}\Omega(N,E,V))}{\Delta S}\right|_{\bar{E}}=-\frac{\eta T}{P}=\text{const.}$$

$$\beta=\frac{1}{kT}\qquad\qquad \eta=\frac{P}{kT}$$

$$\left.\begin{aligned}&\xi=\left.\frac{\partial \mathrm{Ln}\Omega(N,V,E)}{\partial N}\right|_{V,E}\\ &\left.\frac{\partial S}{\partial V}\right|_{N,E}=-\frac{\mu}{T}\\ &S=k\mathrm{Ln}\Omega(N,V,E)\end{aligned}\right\}\Rightarrow \xi=-\frac{\mu}{kT}$$

$$\begin{pmatrix}\beta\\ \eta\\ \xi\end{pmatrix}=\begin{pmatrix}\left.\frac{\partial \mathrm{Ln}\Omega(N,V,E)}{\partial E}\right|_{N,V}\\ \left.\frac{\partial \mathrm{Ln}\Omega(N,V,E)}{\partial V}\right|_{N,E}\\ \left.\frac{\partial \mathrm{Ln}\Omega(N,V,E)}{\partial N}\right|_{V,E}\end{pmatrix}=\begin{pmatrix}\frac{1}{kT}\\ \frac{P}{kT}\\ -\frac{\mu}{kT}\end{pmatrix}$$

تمرین: شرایط پایداری را در رهیافت ترمودینامیکی با استفاده از بسط تیلور آنتروپی و در نظر گرفتن جملات مرتبه دوم مورد بررسی قرار دهید. (راهنمایی: به کتاب رایشل مراجعه نمایید.)